STAT 3202: Homework 01

Autumn 2018, OSU

Due: Friday, August 31

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

The Dormouse, Garfield, and Snorlax are three notorious sleepers. Since none of them exist in the same fictional universe, each sleeper's sleep is independent of the others.

- The Dormouse's sleep follows a normal distribution with a mean of 10 hours and a standard deviation of 2 hours.
- Garfield's sleep follows a normal distribution with a mean of 12 hours and a standard deviation of 2 hours.
- Snorlax's sleep follows a normal distribution with a mean of 14 hours and a standard deviation of 1 hour.

Calculate the probability that on some randomly chosen night, this trio's sleep averages more than 15 hours.

Exercise 2

Suppose that $\text{E}\left[\hat{\theta}_1\right] = \text{E}\left[\hat{\theta}_2\right] = \theta$, $\text{Var}\left[\hat{\theta}_1\right] = \sigma_1^2$, $\text{Var}\left[\hat{\theta}_2\right] = \sigma_2^2$, and $\text{Cov}\left[\hat{\theta}_1, \hat{\theta}_2\right] = \sigma_{12}$. Consider the unbiased estimator

$$\hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2.$$

What value should be chosen for the constant a in order to minimize the variance and thus mean squared error of $\hat{\theta}_3$ as an estimator of θ?

Exercise 3

Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution with density

$$f(x) = \frac{3x^2}{\beta^3}, 0 < x < \beta.$$

In order to estimate β, consider the estimator

$$\frac{4}{3}\bar{X}.$$

Calculate the mean squared error of this estimator.

Hint: You will first need to calculate the expected value and variance of X. Then calculate the bias and variance of the proposed estimator.

Exercise 4

Suppose that the number of accidents per week for a particular brand of electric scooters follows a Poisson distribution with mean λ. A random sample, $Y_1, Y_2, \ldots, Y_n$ of observations on the weekly number of accidents is available. The medical costs for these accidents (in $1,000s of dollars) is $C = 4Y + Y^2$.

Given that $\mathrm{E}[\bar{Y}] = \lambda$ and $\mathrm{E}[C] = 5\lambda + \lambda^2$, find a function of $Y_1, Y_2, \ldots, Y_n$ that is an unbiased estimator for $\mathrm{E}[C]$.

Hint: This estimator will be of the form $a\bar{Y} + b\bar{Y}^2$.

Exercise 5

Suppose that X_1, X_2, X_3 denote a random sample from a normal distribution with an unknown mean μ and a variance of 1. That is,

$$X_i \sim N(\mu, \sigma_1^2 = 1).$$

Consider two estimators,

$$\hat{\mu}_1 = \frac{1}{3}X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3$$

and

$$\hat{\mu}_2 = \frac{1}{9}X_1 + \frac{1}{9}X_2 + \frac{1}{9}X_3.$$

For what values of μ does $\hat{\mu}_2$ obtain a lower MSE than $\hat{\mu}_1$, if any?
