

STAT 3202: Homework 02

Autumn 2018, OSU

Due: Friday, September 7

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Suppose that X_1, X_2, \dots, X_n and Y_1, Y_2, \dots, Y_n are independent random samples from populations with the same mean μ and variances σ_X^2 and σ_Y^2 , respectively.

That is,

$$X_i \sim N(\mu, \sigma_X^2)$$

$$Y_i \sim N(\mu, \sigma_Y^2)$$

Show that $\frac{2\bar{X} + 3\bar{Y}}{5}$ is a consistent estimator of μ .

Exercise 2

Let Y_1, Y_2, \dots, Y_n denote a random sample from the probability density function

$$f(y | \theta) = \theta y^{\theta-1}, \quad 0 < y < 1, \theta > 0.$$

Show that \bar{Y} is a consistent estimator of $\frac{\theta}{\theta + 1}$.

Exercise 3

Consider two binomial random variables Y_1 and Y_2 . In particular,

$$Y_1 \sim \text{binom}(n, p_1)$$

$$Y_2 \sim \text{binom}(n, p_2)$$

Propose and justify a consistent estimator for $p_1 - p_2$.

Exercise 4

If Y_1, Y_2, \dots, Y_n denote a random sample from a geometric distribution with parameter p , show that \bar{Y} is sufficient for p .

Exercise 5

If X_1, X_2, \dots, X_n denote a random sample from the probability density function

$$f(x | \beta) = \frac{1}{2\beta^3} x^2 e^{-x/\beta}, \quad x > 0, \beta > 0.$$

Show that \bar{X} is sufficient for β .
