STAT 3202: Homework 03

Autumn 2018, OSU

Due: Friday, September 14

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function

$$f(x \mid \theta) = \left(\theta^2 + \theta\right) x^{\theta - 1}(1 - x), \quad 0 < x < 1, \theta > 0.$$

Obtain a method of moments **estimator** for θ, $\tilde{\theta}$. Calculate an **estimate** using this *estimator* when

$$x_1 = 0.50, \ x_2 = 0.75, \ x_3 = 0.80, \ x_4 = 0.25.$$

Exercise 2

Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed uniform random variables on the interval $(0, 4\lambda)$.

Obtain a method of moments **estimator** for λ, $\tilde{\lambda}$. Calculate the variance of this estimator. (Your answer will be a function of the sample size n and λ.)

Exercise 3

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x \mid \lambda) = \lambda x^{\lambda - 1}, \quad 0 < x < 1, \ \lambda > 0$$

Obtain the maximum likelihood **estimator** of λ, $\hat{\lambda}$. Calculate an **estimate** using this maximum likelihood **estimator** when

$$x_1 = 0.10, \ x_2 = 0.20, \ x_3 = 0.30, \ x_4 = 0.40.$$

Exercise 4

Let $X_1, X_2, \ldots, X_n$ denote independent and identically distributed uniform random variables on the interval $[0, 3\beta]$.

Obtain the maximum likelihood **estimator** for β, $\hat{\beta}$. Use this **estimator** to provide an **estimate** of Var$[X]$ when

$$x_1 = 1.3,\ x_2 = 5.7,\ x_3 = 2.2.$$

Exercise 5

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x \mid \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0,\ \theta > 0$$

Obtain the maximum likelihood **estimator** of θ, $\hat{\theta}$. Use this maximum likelihood **estimator** to obtain an **estimate** of

$$P[X > 4]$$

when

$$x_1 = 0.50,\ x_2 = 1.50,\ x_3 = 4.00,\ x_4 = 3.00.$$
