

STAT 3202: Practice 02

Autumn 2018, OSU

Exercise 1

Let X_1, X_2, \dots, X_n be iid $N(\theta, 1)$ and consider $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$. Show that \bar{X}_n is a consistent estimator of θ .

Exercise 2

Suppose that X_1, X_2, \dots, X_n are an iid sample from the distribution

$$f(x; \theta) = \frac{1}{2}(1 + \theta x), \quad -1 < x < 1, -1 < \theta < 1.$$

Show that $3\bar{X}_n$ is a consistent estimator of θ .

Exercise 3

Let Y_1, Y_2, \dots, Y_n be a random sample such that

- $E[Y_i] = \mu$
- $\text{Var}[Y_i] = \sigma^2$.

Suggest a consistent estimator for μ^2 .

Exercise 4

Let X_1, X_2, \dots, X_n be iid $N(\mu_X, \sigma_X^2)$. Also, let Y_1, Y_2, \dots, Y_n be iid $N(\mu_Y, \sigma_Y^2)$.

Suggest a consistent estimator for $\mu_X - \mu_Y$.

Exercise 5

Let X_1, X_2, \dots, X_n be iid $N(\mu_X, \sigma^2)$. Also, let Y_1, Y_2, \dots, Y_n be iid $N(\mu_Y, \sigma^2)$. Note that both distributions have the same variance.

Show that

$$\frac{\sum_{i=1}^n (X_i - \bar{X})^2 + \sum_{i=1}^n (Y_i - \bar{Y})^2}{2n - 2}$$

is a consistent estimator for σ^2 .

Hint: Note that

$$\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2} \sim \chi_{n-1}^2$$

$$\frac{\sum_{i=1}^n (Y_i - \bar{Y})^2}{\sigma^2} \sim \chi_{n-1}^2$$

Also, recall that, if $W \sim \chi_k^2$, then $E[W] = k$ and $\text{Var}[W] = 2k$.

Exercise 6

Let Y_1, Y_2, \dots, Y_n be iid observations from a Poisson distribution with parameter λ . Show that $U = \sum_{i=1}^n Y_i$ is sufficient for λ .

Exercise 7

Let X_1, X_2, \dots, X_n be iid observations from a distribution with density

$$f(x | \theta) = \frac{\theta}{(1+x)^{\theta+1}}, \quad 0 < \theta < \infty, 0 < x < \infty$$

Find a sufficient statistic for θ .

Exercise 8

Let X_1, X_2, \dots, X_n be iid observations from a normal distribution with a unknown mean, μ , and known variance $\sigma^2 = 9$.

Show that $\sum_{i=1}^n X_i$ is a sufficient statistic for μ , then use this statistic to create an estimator that is both unbiased and sufficient for estimating μ .

Exercise 9

Let Y_1, Y_2, \dots, Y_n be iid observations from a distribution with density

$$f(y | \beta) = \frac{y}{\beta} \cdot \exp\left(\frac{-y^2}{2\beta}\right), \quad y \geq 0, \beta > 0$$

Find a sufficient statistic for β .

Exercise 10

Let X_1, X_2, \dots, X_n be iid observations from a distribution with density

$$f(x | \alpha, \beta) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}, \quad x \geq 0, \alpha > 0, \beta > 0$$

Let α be a known constant and β be unknown. Find a sufficient statistic for β .
