

STAT 3202: Practice 06

Autumn 2018, OSU

Exercise 1

Consider a random sample of size $n = 50$. The sample mean and standard deviation are:

- $\bar{x} = 5$
- $s = 3$

Use this sample to test $H_0: \mu = 4$ vs $H_1: \mu > 4$.

Report:

- The **test statistic**
- The **critical value** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $z = 2.36$
 - **Critical value:** 1.645
 - Reject if $z > 1.645$
 - **Decision:** Reject H_0
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Exercise 2

Consider a random sample of size $n = 100$. The sample mean and standard deviation are:

- $\bar{x} = -0.5$
- $s = 2$

Use this sample to test $H_0: \mu = 0$ vs $H_1: \mu \neq 0$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic:** $z = -2.5$
 - **P-value:** 0.0124
 - **Decision:** Fail to reject H_0
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Exercise 3

Consider two independent random samples.

Sample 1, from Population X :

- $n_x = 50$
- $\bar{x} = 5$
- $s_x = 2$

Sample 2, from Population Y :

- $n_y = 45$
- $\bar{y} = 6$
- $s_y = 3$

Use these samples to test $H_0: \mu_x = \mu_y$ vs $H_1: \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $z = -1.89$
 - **P-value:** 0.0588
 - **Decision:** Fail to reject H_0
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Exercise 4

Consider two independent random samples.

Sample 1, from Population X :

- $n_x = 75$
- $\bar{x} = 13$
- $s_x = 5$

Sample 2, from Population Y :

- $n_y = 100$
- $\bar{y} = 11$
- $s_y = 6$

Use these samples to test $H_0: \mu_x = \mu_y$ vs $H_1: \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **critical values** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $z = 2.40$
 - **Critical values:** $-1.960, 1.960$
 - Reject if $z > 1.960$
 - Reject if $z < -1.960$
 - **Decision:** Reject H_0
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Exercise 5

Consider a random sample of size $n = 50$ from a dichotomous population. The sample proportion of the “success” class is

- $\hat{p} = 0.58$

Use this sample to test $H_0: p = 0.50$ vs $H_1: p \neq 0.50$.

Report:

- The **test statistic**
- The **critical value** when $\alpha = 0.01$.
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic:** $z = 1.13$
 - **Critical values:** $-2.576, 2.576$
 - Reject if $z > 2.576$
 - Reject if $z < -2.576$
 - **Decision:** Fail to reject H_0
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Exercise 6

Consider a random sample of size $n = 100$ from a dichotomous population. The sample proportion of the “success” class is

- $\hat{p} = 0.81$

Use this sample to test $H_0: p = 0.70$ vs $H_1: p > 0.70$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $z = 2.40$
- **P-value:** 0.0082

- **Decision:** Reject H_0
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Exercise 7

Consider two independent random samples from dichotomous populations.

Sample 1, from Population X :

- $n_x = 80$
- $\hat{p}_x = 0.70$

Sample 2, from Population Y :

- $n_y = 90$
- $\hat{p}_y = 0.79$

Use these samples to test $H_0: p_x = p_y$ vs $H_1: p_x \neq p_y$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.10$.

Solution

- **Test statistic:** $z = -1.39$
– $\hat{p} = 0.75$
 - **P-value:** 0.1646
 - **Decision:** Fail to reject H_0
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Exercise 8

Consider two independent random samples from dichotomous populations.

Sample 1, from Population X :

- $n_x = 100$
- $\hat{p}_x = 0.39$

Sample 2, from Population Y :

- $n_y = 200$
- $\hat{p}_y = 0.51$

Use these samples to test $H_0: p_x = p_y$ vs $H_1: p_x \neq p_y$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic:** $z = -1.96$
– $\hat{p} = 0.47$
 - **P-value:** 0.05
 - **Decision:** Fail to reject H_0
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Exercise 9

Consider a random sample of size $n = 12$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = 5$
- $s = 2$

Use this sample to test $H_0: \mu = 4$ vs $H_1: \mu > 4$.

Report:

- The **test statistic**
- The **critical value** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $t = 1.76$
– $df = 11$
 - **Critical value:** 1.796
– Reject if $t > 1.796$
 - **Decision:** Fail to reject H_0
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Exercise 10

Consider a random sample of size $n = 8$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = -1.2$
- $s = 3$

Use this sample to test $H_0: \mu = 0$ vs $H_1: \mu < 0$.

Report:

- The **test statistic**
- The **critical value** when $\alpha = 0.10$.
- A **decision** when $\alpha = 0.10$.

Solution

- **Test statistic:** $t = -1.13$
 - $df = 7$
 - **Critical value:** -1.415
 - Reject if $t < -1.415$
 - **Decision:** Fail to reject H_0
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Exercise 11

Consider a random sample of size $n = 12$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = 5$
- $s = 2.3$

Use this sample to test $H_0: \sigma = 2$ vs $H_1: \sigma > 2$.

Report:

- The **test statistic**
- The **critical value** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $X^2 = 14.5475$
 - $df = 11$
 - **Critical value:** 19.6751
 - Reject if $X^2 > 19.6751$
 - **Decision:** Fail to reject H_0
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Exercise 12

Consider a random sample of size $n = 22$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = 7$
- $s = 5.6$

Use this sample to test $H_0: \sigma = 5$ vs $H_1: \sigma > 5$.

Report:

- The **test statistic**
- The **critical value** when $\alpha = 0.01$.
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic:** $X^2 = 26.3424$
 - $df = 21$
 - **Critical value:** 38.9321
 - Reject if $X^2 > 38.9321$
 - **Decision:** Fail to reject H_0
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Exercise 13

Consider two independent random samples. Assume both populations are normal and that their variances are equal.

Sample 1, from Population X :

- $n_x = 10$
- $\bar{x} = 5$
- $s_x = 2$

Sample 2, from Population Y :

- $n_y = 12$
- $\bar{y} = 6$
- $s_y = 1.5$

Use these samples to test $H_0: \mu_x = \mu_y$ vs $H_1: \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **critical values** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic:** $t = -2.68$
 - $s_p = \sqrt{3.0375}$
 - $df = 20$
 - **Critical values:** $-2.086, 2.086$
 - Reject if $t > 2.086$
 - Reject if $t < -2.086$
 - **Decision:** Reject H_0
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Exercise 14

Consider two independent random samples. Assume both populations are normal and that their variances are equal.

Sample 1, from Population X :

- $n_x = 14$
- $\bar{x} = 48, 530$

- $s_x = 780$

Sample 2, from Population Y:

- $n_y = 11$
- $\bar{y} = 47,620$
- $s_y = 750$

Use these samples to test $H_0: \mu_x = \mu_y$ vs $H_1: \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **critical values** when $\alpha = 0.01$.
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic:** $t = 2.944$
 - $s_p = 767.1$
 - $df = 23$
 - **Critical values:** $-2.807, 2.807$
 - Reject if $t > 2.807$
 - Reject if $t < -2.807$
 - **Decision:** Reject H_0
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