STAT 3202: Practice 10

Autumn 2018, OSU

Exercise 1

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 5, \beta = 5)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 20$
- Number of "successes" $\sum x_i = 15$

Use the given prior and the observed data to calculate a Bayes' estimate of θ. (Use the posterior mean.)

Exercise 2

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 50, \beta = 20)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 40$
- Number of "successes" $\sum x_i = 32$

Use the given prior and the observed data to calculate a Bayes' estimate of θ. (Use the posterior mean.)

Exercise 3

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 10, \beta = 60)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 5$
- Number of "successes" $\sum x_i = 2$

Use the given prior and the observed data to calculate a Bayes' estimate of θ. (Use the posterior mean.)

Exercise 4

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 10, \beta = 60)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 5$
- Number of "successes" $\sum x_i = 2$

Use the given prior and the observed data to calculate a 99% credible interval

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Exercise 5

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 4, \beta = 4)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 5$
- Number of "successes" $\sum x_i = 2$

Use the given prior and the observed data to calculate a 90% credible interval

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Exercise 6

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 10, \beta = 4)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 50$
- Number of "successes" $\sum x_i = 2$

Use the given prior and the observed data to calculate a 95% credible interval

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Exercise 7

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 10, \beta = 4)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 50$
- Number of "successes" $\sum x_i = 20$

Use the given prior and the observed data to test $H_0 : \theta > 0.50$ vs $H_1 : \theta \leq 0.50$

Exercise 8

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 10, \beta = 10)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 20$
- Number of "successes" $\sum x_i = 5$

Use the given prior and the observed data to test $H_0 : 0.25 < \theta < 0.50$ vs $H_1 : \theta \leq 0.25, \theta \geq 0.50$

Exercise 9

Consider the following model,

- Prior: $\theta \sim \text{Beta}(\alpha = 3, \beta = 3)$
- Likelihood: $X_1, X_1, \ldots, X_n \sim \text{Bern}(\theta)$
- Posterior: $\theta \mid X_1, X_1, \ldots, X_n \sim ?$

and observed data with statistics,

- Sample size: $n = 100$
- Number of "successes" $\sum x_i = 70$

Use the given prior and the observed data to test $H_0 : \theta > 0.80$ vs $H_1 : \theta \leq 0.80$
