STAT 3202 – (Some) Final Exam Practice

Autumn 2018

1. Let $\tau > 0$ and let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x;\tau) = \frac{\tau^5}{8} x^{14} e^{-\tau x^3}, \qquad x > 0.$$

Obtain the maximum likelihood estimator of τ, $\hat{\tau}$.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x) = \frac{2(\theta - x)}{\theta^2} \qquad 0 < x < \theta \qquad \theta > 0.$$

The method of moments estimator of θ, $\tilde{\theta}$, is given by

$$\tilde{\theta} = 3 \cdot \overline{X} = 3 \cdot \frac{1}{n} \cdot \sum_{i=1}^{n} X_i$$

a) Is $\tilde{\theta}$ an unbiased estimator for θ?

b) Find $\mathrm{Var}(\tilde{\theta})$.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the distribution with probability density function

$$f(x;\lambda) = \frac{2\sqrt{\lambda}}{\sqrt{\pi}} e^{-\lambda x^2}, \quad x > 0, \qquad \lambda > 0.$$

a) Obtain the maximum likelihood estimator of λ, $\hat{\lambda}$.

b) Suppose $n = 4$, and $x_1 = 0.2$, $x_2 = 0.6$, $x_3 = 1.1$, $x_4 = 1.7$.
Find the maximum likelihood estimate of λ.

4. Let $\lambda > 0$ and let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x;\lambda) = \frac{\lambda^2}{2} e^{-\lambda\sqrt{x}}, \qquad x > 0, \qquad \text{zero otherwise.}$$

a) Find the maximum likelihood estimator of λ, $\hat{\lambda}$.

b) Suppose $n = 4$, and $x_1 = 0.81$, $x_2 = 1.96$, $x_3 = 0.36$, $x_4 = 0.09$.
Find the maximum likelihood estimate of λ, $\hat{\lambda}$.

5. Several employees of *Al's Building Construction* complain that the variance of the employees' monthly salary amounts is too large. In a random sample of 10 employees, the sample standard deviation of the monthly salary amounts was $210.

a) Use $\alpha = 0.05$ to test $H_0: \sigma \leq 150$ vs. $H_1: \sigma > 150$. Report the value of the test statistic, the critical value(s), and state your decision.

b) Using the chi-square distribution table only, what is the p-value of the test in part (a)? (You may give a range.)

6. The service time in queues should not have a large variance; otherwise, the queue tends to build up. A bank regularly checks the service times of its tellers to determine their variance. A random sample of 22 service times (in minutes) gives $s^2 = 8$. Assume the service times are normally distributed.

a) Find a 95% confidence interval for the overall variance of service time at the bank.

b) Find the two one-sided 95% confidence intervals for the overall variance of service time at the bank.

7. In a random sample of 200 students who took Exam P in 2017, 74 successfully passed the exam. (Exam P [Probability] is the first Actuarial Science exam.) Suppose also that in a random sample of 150 students who took Exam FM in 2017, 45 successfully passed the exam. (Exam FM [Financial Mathematics] is the second Actuarial Science exam.)

a) Construct a 95% confidence interval for the difference between the overall proportions of students who passed Exam P and Exam FM in 2017.

b) At a 10% level of significance, test whether Exam P had a higher success rate than Exam FM in 2017. Find the p-value of this test.

8. A random sample of 9 adult white rhinos had the sample mean weight of 5,100 pounds and the sample standard deviation of 450 pounds. A random sample of 16 adult hippos had the sample mean weight of 3,300 pounds and the sample standard deviation of 400 pounds. Assume that the two populations are approximately normally distributed. Construct a 95% confidence interval for the difference between their overall average weights of adult white rhinos and adult hippos. Assume that the overall standard deviations are equal.