STAT 3202: Homework 02

Spring 2019, OSU

Due: Friday, January 25

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Suppose that $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples from populations with the same mean μ and variances σ_X^2 and σ_Y^2, respectively.

That is,

$$X_i \sim \text{N}(\mu, \sigma_X^2)$$
$$Y_i \sim \text{N}(\mu, \sigma_Y^2)$$

Show that $\dfrac{2\bar{X} + 3\bar{Y}}{5}$ is a consistent estimator of μ.

Exercise 2

Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \theta y^{\theta - 1}, \quad 0 < y < 1, \theta > 0.$$

Show that $\bar{Y}$ is a consistent estimator of $\dfrac{\theta}{\theta + 1}$.

Exercise 3

Consider two binomial random variables Y_1 and Y_2. In particular,

$$Y_1 \sim \text{binom}(n, p_1)$$
$$Y_2 \sim \text{binom}(n, p_2)$$

Propose and justify a consistent estimator for $p_1 - p_2$.

Exercise 4

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function

$$f(x \mid \theta) = \left(\theta^2 + \theta\right) x^{\theta-1}(1-x), \quad 0 < x < 1, \theta > 0.$$

Obtain a method of moments **estimator** for θ, $\tilde{\theta}$. Calculate an **estimate** using this *estimator* when

$$x_1 = 0.50,\ x_2 = 0.75,\ x_3 = 0.85,\ x_4 = 0.25.$$

Exercise 5

Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed uniform random variables on the interval $(0, 4\lambda)$.

Obtain a method of moments **estimator** for λ, $\tilde{\lambda}$. Calculate the mean squared error of this estimator when estimating λ. (Your answer will be a function of the sample size n and λ.)
