STAT 3202: Homework 03

Spring 2019, OSU

Due: Friday, February 1

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x \mid \lambda) = \lambda x^{\lambda - 1}, \quad 0 < x < 1, \ \lambda > 0$$

Obtain the maximum likelihood **estimator** of λ, $\hat{\lambda}$. Calculate an **estimate** using this maximum likelihood **estimator** when

$$x_1 = 0.10, \ x_2 = 0.20, \ x_3 = 0.30, \ x_4 = 0.70.$$

Exercise 2

In genetics, single nucleotide polymorphisms (SNPs) are locations in the (human) genome that exhibit variation across the population. SNPs cause the differences we see in traits such as hair color. Each SNP typically has two possible alleles – say A and a – and each person's genotype at the SNP is either AA, Aa, or aa, where one allele comes from the person's mother and one from the father. Let X be the number of A alleles at a particular SNP, and suppose we collect a random sample of people from some population. Under some assumptions (such as "random mating" and "no selection") we may assume that

$$X_1, X_2, \ldots, X_n \stackrel{\text{iid}}{\sim} \text{Binom}(2, p),$$

where p is called the *allele frequency* of allele A. What is the maximum likelihood **estimator** of p? What is the maximum likelihood **estimate** of the allele frequency of allele A if our sample consists of five people with genotypes

$$AA, aa, Aa, aa, Aa$$

at this particular SNP?

Exercise 3

Consider two corn varieties, A and B, both grown in the Morrow Plots. (Illinois is very serious about their corn. Rumor has it, if a student is found trespassing in the Morrow Plots, they will be expelled…)

Suppose that $X_1, X_2, \ldots, X_n$, representing yields per acre for corn variety A, constitute a random sample from a normal distribution with mean μ_1 and variance θ. (In more usual notation, $\theta = \sigma^2$, but we are using θ here to make the notation easier in this problem.) Also, $Y_1, Y_2, \ldots, Y_m$, representing yields for corn variety B, constitute a random sample from a normal distribution with mean μ_2 and variance θ. If the X_i and Y_j are all mutually independent, find the maximum likelihood **estimator** for the common variance θ. Assume that μ_1 and μ_2 are **known.**

Exercise 4

Let $X_1, X_2, \ldots, X_n$ denote independent and identically distributed uniform random variables on the interval $[0, 3\beta]$.

Obtain the maximum likelihood **estimator** for β, $\hat{\beta}$. Use this **estimator** to provide an **estimate** of $\text{Var}[X]$ when

$$x_1 = 1.3, \ x_2 = 3.9, \ x_3 = 2.2.$$

Exercise 5

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x \mid \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0, \ \theta > 0$$

Obtain the maximum likelihood **estimator** of θ, $\hat{\theta}$. Use this maximum likelihood **estimator** to obtain an **estimate** of

$$P[X > 4]$$

when

$$x_1 = 0.50, \ x_2 = 1.50, \ x_3 = 4.00, \ x_4 = 3.00.$$
