STAT 3202: Practice 02

Spring 2019, OSU

Exercise 1

Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, 1)$ and consider $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $\bar{X}_n$ is a consistent estimator of θ.

Exercise 2

Suppose that $X_1, X_2, \ldots, X_n$ are an iid sample from the distribution

$$f(x; \theta) = \frac{1}{2}(1 + \theta x), \quad -1 < x < 1, -1 < \theta < 1.$$

Show that $3\bar{X}_n$ is a consistent estimator of θ.

Exercise 3

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample such that

- $\text{E}[Y_i] = \mu$
- $\text{Var}[Y_i] = \sigma^2$.

Suggest a consistent estimator for μ^2.

Exercise 4

Let $X_1, X_2, \ldots, X_n$ be iid $N(\mu_X, \sigma_X^2)$. Also, let $Y_1, Y_2, \ldots, Y_n$ be iid $N(\mu_Y, \sigma_Y^2)$.

Suggest a consistent estimator for $\mu_X - \mu_Y$.

Exercise 5

Let $X_1, X_2, \ldots, X_n$ be iid $N(\mu_X, \sigma^2)$. Also, let $Y_1, Y_2, \ldots, Y_n$ be iid $N(\mu_Y, \sigma^2)$. Note that both distributions have the same variance.

Show that

$$\frac{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2+\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)^2}{2n-2}$$

is a consistent estimator for σ^2.

Hint: Note that

$$\frac{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}{\sigma^2}\sim\chi_{n-1}^2$$

$$\frac{\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)^2}{\sigma^2}\sim\chi_{n-1}^2$$

Also, recall that, if $W\sim\chi_k^2$, then $\mathrm{E}[W]=k$ and $\mathrm{Var}[W]=2k$.

Exercise 6

Let $Y_1,Y_2,\ldots,Y_n$ be iid observations from a Poisson distribution with parameter λ. Show that $U=\sum_{i=1}^{n}Y_i$ is sufficient for λ.

Exercise 7

Let $X_1,X_2,\ldots,X_n$ be iid observations from a distribution with density

$$f(x\mid\theta)=\frac{\theta}{(1+x)^{\theta+1}},\quad 0<\theta<\infty,0<x<\infty$$

Find a sufficient statistic for θ.

Exercise 8

Let $X_1,X_2,\ldots,X_n$ be iid observations from a normal distribution with a unknown mean, μ, and known variance $\sigma^2=9$.

Show that $\sum_{i=1}^{n}X_i$ is a sufficient statistic for μ, then use this statistic to create an estimator that is both unbiased and sufficient for estimating μ.

Exercise 9

Let $Y_1, Y_2, \ldots, Y_n$ be iid observations from a distribution with density

$$f(y \mid \beta) = \frac{y}{\beta} \cdot \exp\left(\frac{-y^2}{2\beta}\right), \quad y \geq 0, \beta > 0$$

Find a sufficient statistic for β.

Exercise 10

Let $X_1, X_2, \ldots, X_n$ be iid observations from a distribution with density

$$f(x \mid \alpha, \beta) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}, \quad x \geq 0, \alpha > 0, \beta > 0$$

Let α be a known constant and β be unknown. Find a sufficient statistic for β.
