STAT 3202: Practice 03

Spring 2019, OSU

Exercise 1

Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \text{Poisson}(\lambda)$. That is

$$f(x \mid \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots \quad \lambda > 0$$

(a) Obtain a method of moments **estimator** for λ, $\tilde{\lambda}$. Calculate an **estimate** using this *estimator* when

$$x_1 = 1, \ x_2 = 2, \ x_3 = 4, \ x_4 = 2.$$

(b) Find the maximum likelihood **estimator** for λ, $\hat{\lambda}$. Calculate an **estimate** using this *estimator* when

$$x_1 = 1, \ x_2 = 2, \ x_3 = 4, \ x_4 = 2.$$

(c) Find the maximum likelihood **estimator** of $P[X = 4]$, call it $\hat{P}[X = 4]$. Calculate an **estimate** using this *estimator* when

$$x_1 = 1, \ x_2 = 2, \ x_3 = 4, \ x_4 = 2.$$

Exercise 2

Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\theta, \sigma^2)$.

Find a method of moments **estimator** for the *parameter vector* $\left(\theta, \sigma^2\right)$.

Exercise 3

Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(1, \sigma^2)$.

Find a method of moments **estimator** of σ^2, call it $\tilde{\sigma}^2$.

Exercise 4

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with pdf

$$f(x \mid \theta) = \frac{1}{\theta} x^{(1-\theta)/\theta}, \quad 0 < x < 1,\ 0 < \theta < \infty$$

(a) Find the maximum likelihood **estimator** of θ, call it $\hat{\theta}$. Calculate an **estimate** using this *estimator* when

$$x_1 = 0.10,\ x_2 = 0.22,\ x_3 = 0.54,\ x_4 = 0.36.$$

(b) Obtain a method of moments **estimator** for θ, $\tilde{\theta}$. Calculate an **estimate** using this *estimator* when

$$x_1 = 0.10,\ x_2 = 0.22,\ x_3 = 0.54,\ x_4 = 0.36.$$

Exercise 5

Let $X_1, X_2, \ldots, X_n$ iid from a population with pdf

$$f(x \mid \theta) = \frac{\theta}{x^2}, \quad 0 < \theta \le x$$

Obtain the maximum likelihood **estimator** for θ, $\hat{\theta}$.

Exercise 6

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x, \alpha) = \alpha^{-2} x e^{-x/\alpha}, \quad x > 0,\ \alpha > 0$$

(a) Obtain the maximum likelihood **estimator** of α, $\hat{\alpha}$. Calculate the **estimate** when

$$x_1 = 0.25,\ x_2 = 0.75,\ x_3 = 1.50,\ x_4 = 2.5,\ x_5 = 2.0.$$

(b) Obtain the method of moments **estimator** of α, $\tilde{\alpha}$. Calculate the **estimate** when

$$x_1 = 0.25,\ x_2 = 0.75,\ x_3 = 1.50,\ x_4 = 2.5,\ x_5 = 2.0.$$

Hint: Recall the probability density function of an exponential random variable.

$$f(x \mid \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0,\ \theta > 0$$

Note that, the moments of this distribution are given by

$$E[X^k] = \int_0^\infty \frac{x^k}{\theta} e^{-x/\theta} = k! \cdot \theta^k.$$

This hint will also be useful in the next exercise.

Exercise 7

Let $X_1, X_2, \dots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x \mid \beta) = \frac{1}{2\beta^3} x^2 e^{-x/\beta}, \quad x > 0,\ \beta > 0$$

(a) Obtain the maximum likelihood **estimator** of β, $\hat{\beta}$. Calculate the **estimate** when

$$x_1 = 2.00,\ x_2 = 4.00,\ x_3 = 7.50,\ x_4 = 3.00.$$

(b) Obtain the method of moments **estimator** of β, $\tilde{\beta}$. Calculate the **estimate** when

$$x_1 = 2.00,\ x_2 = 4.00,\ x_3 = 7.50,\ x_4 = 3.00.$$

Exercise 8

Let $Y_1, Y_2, \dots, Y_n$ be a random sample from a distribution with pdf

$$f(y \mid \alpha) = \frac{2}{\alpha} \cdot y \cdot \exp\left\{-\frac{y^2}{\alpha}\right\},\ y > 0,\ \alpha > 0.$$

(a) Find the maximum likelihood **estimator** of α.

(b) Let $Z_1 = Y_1^2$. Find the distribution of Z_1. Is the MLE for α an unbiased estimator of α?

Exercise 9

Let X be a single observation from a Binom(n, p), where p is an unknown parameter. (In this case, we will consider n known.)

(a) Find the maximum likelihood **estimator** (MLE) of p.

(b) Suppose you roll a 6-sided die 40 times and observe eight rolls of a 6. What is the maximum likelihood **estimate** of the probability of observing a 6?

(c) Using the same observed data, suppose you now plan to perform a second experiment with the same die, and will roll the die 5 more times. What is the maximum likelihood **estimate** of the probability that you will observe no 6's in this next experiment?

Exercise 10

Suppose that a random variable X follows a discrete distribution, which is determined by a parameter θ which can take *only two values*, $\theta = 1$ or $\theta = 2$. The parameter θ is unknown.

- If $\theta = 1$, then X follows a Poisson distribution with parameter $\lambda = 2$.
- If $\theta = 2$, then X follows a Geometric distribution with parameter $p = \frac{1}{4}$.

Now suppose we observe $X = 3$. Based on this data, what is the maximum likelihood **estimate** of θ?

Exercise 11

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with pdf

$$f(y \mid \theta) = \frac{2\theta^2}{y^3}, \quad \theta \leq y < \infty$$

Find the maximum likelihood **estimator** of θ..
