STAT 3202: Practice 06

Spring 2019, OSU

Exercise 1

Consider a random sample of size $n = 50$. The sample mean and standard deviation are:

- $\bar{x} = 5$
- $s = 3$

Use this sample to test $H_0: \ \mu = 4$ vs $H_1: \ \mu > 4$.

Report:

- The **test statistic**
- The **crictical value** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $z = 2.36$
- **Crictical value**: 1.645
  - Reject if $z > 1.645$
- **Decision**: Reject H_0

Exercise 2

Consider a random sample of size $n = 100$. The sample mean and standard deviation are:

- $\bar{x} = -0.5$
- $s = 2$

Use this sample to test $H_0: \ \mu = 0$ vs $H_1: \ \mu \neq 0$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic**: $z = -2.5$
- **P-value**: 0.0124
- **Decision**: Fail to reject H_0

Exercise 3

Consider two independent random samples.

Sample 1, from Population X:

- $n_x = 50$
- $\bar{x} = 5$
- $s_x = 2$

Sample 2, from Population Y:

- $n_y = 45$
- $\bar{y} = 6$
- $s_y = 3$

Use these samples to test $H_0: \ \mu_x = \mu_y$ vs $H_1: \ \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $z = -1.89$
- **P-value**: 0.0588
- **Decision**: Fail to reject H_0

Exercise 4

Consider two independent random samples.

Sample 1, from Population X:

- $n_x = 75$
- $\bar{x} = 13$
- $s_x = 5$

Sample 2, from Population Y:

- $n_y = 100$
- $\bar{y} = 11$
- $s_y = 6$

Use these samples to test $H_0: \ \mu_x = \mu_y$ vs $H_1: \ \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **crictical values** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $z = 2.40$
- **Crictical values**: $-1.960, 1.960$
  - Reject if $z > 1.960$
  - Reject if $z < -1.960$
- **Decision**: Reject H_0

Exercise 5

Consider a random sample of size $n = 50$ from a dichotomous population. The sample proportion of the "success" class is

- $\hat{p} = 0.58$

Use this sample to test $H_0: \; p = 0.50 \;$ vs $\; H_1: \; p \neq 0.50$.

Report:

- The **test statistic**
- The **crictical value** when $\alpha = 0.01$.
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic**: $z = 1.13$
- **Crictical values**: $-2.576, 2.576$
  - Reject if $z > 2.576$
  - Reject if $z < -2.576$
- **Decision**: Fail to reject H_0

Exercise 6

Consider a random sample of size $n = 100$ from a dichotomous population. The sample proportion of the "success" class is

- $\hat{p} = 0.81$

Use this sample to test $H_0: \; p = 0.70 \;$ vs $\; H_1: \; p > 0.70$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $z = 2.40$
- **P-value**: 0.0082
- **Decision**: Reject H_0

Exercise 7

Consider two independent random samples from dichotomous populations.

Sample 1, from Population X:

- $n_x = 80$
- $\hat{p}_x = 0.70$

Sample 2, from Population Y:

- $n_y = 90$
- $\hat{p}_y = 0.79$

Use these samples to test $H_0: \; p_x = p_y \;$ vs $\; H_1: \; p_x \neq p_y$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.10$.

Solution

- **Test statistic**: $z = -1.39$
    - $\hat{p} = 0.75$
- **P-value**: 0.1646
- **Decision**: Fail to reject H_0

Exercise 8

Consider two independent random samples from dichotomous populations.

Sample 1, from Population X:

- $n_x = 100$
- $\hat{p}_x = 0.39$

Sample 2, from Population Y:

- $n_y = 200$
- $\hat{p}_y = 0.51$

Use these samples to test $H_0: \; p_x = p_y \;$ vs $\; H_1: \; p_x \neq p_y$.

Report:

- The **test statistic**
- The **p-value**
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic**: $z = -1.96$
    - $\hat{p} = 0.47$
- **P-value**: 0.05
- **Decision**: Fail to reject H_0

Exercise 9

Consider a random sample of size $n = 12$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = 5$
- $s = 2$

Use this sample to test $H_0: \; \mu = 4 \;$ vs $\; H_1: \; \mu > 4$.

Report:

- The **test statistic**

- The **crictical value** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $t = 1.76$
  - $df = 11$
- **Crictical value**: 1.796
  - Reject if $t > 1.796$
- **Decision**: Fail to reject H_0

Exercise 10

Consider a random sample of size $n = 8$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = -1.2$
- $s = 3$

Use this sample to test $H_0: \ \mu = 0 \ \text{ vs } \ H_1: \ \mu < 0$.

Report:

- The **test statistic**
- The **crictical value** when $\alpha = 0.10$.
- A **decision** when $\alpha = 0.10$.

Solution

- **Test statistic**: $t = -1.13$
  - $df = 7$
- **Crictical value**: -1.415
  - Reject if $t < -1.415$
- **Decision**: Fail to reject H_0

Exercise 11

Consider a random sample of size $n = 12$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = 5$
- $s = 2.3$

Use this sample to test $H_0: \ \sigma = 2 \ \text{ vs } \ H_1: \ \sigma > 2$.

Report:

- The **test statistic**
- The **crictical value** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $X^2 = 14.5475$
  - $df = 11$

- **Crictical value**: 19.6751
  - Reject if $X^2 > 19.6751$
- **Decision**: Fail to reject H_0

Exercise 12

Consider a random sample of size $n = 22$ from a population that is assumed to be normal. The sample mean and standard deviation are:

- $\bar{x} = 7$
- $s = 5.6$

Use this sample to test $H_0: \ \sigma = 5 \ \text{ vs } \ H_1: \ \sigma > 5$.

Report:

- The **test statistic**
- The **crictical value** when $\alpha = 0.01$.
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic**: $X^2 = 26.3424$
  - $df = 21$
- **Crictical value**: 38.9321
  - Reject if $X^2 > 38.9321$
- **Decision**: Fail to reject H_0

Exercise 13

Consider two independent random samples. Assume both populations are normal and that their variances are equal.

Sample 1, from Population X:

- $n_x = 10$
- $\bar{x} = 5$
- $s_x = 2$

Sample 2, from Population Y:

- $n_y = 12$
- $\bar{y} = 6$
- $s_y = 1.5$

Use these samples to test $H_0: \ \mu_x = \mu_y \ \text{ vs } \ H_1: \ \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **crictical values** when $\alpha = 0.05$.
- A **decision** when $\alpha = 0.05$.

Solution

- **Test statistic**: $t = -2.68$
  - $s_p = \sqrt{3.0375}$
  - $df = 20$
- **Crictical values**: $-2.086, 2.086$
  - Reject if $t > 2.086$
  - Reject if $t < -2.086$
- **Decision**: Reject H_0

Exercise 14

Consider two independent random samples. Assume both populations are normal and that their variances are equal.

Sample 1, from Population X:

- $n_x = 14$
- $\bar{x} = 48,530$
- $s_x = 780$

Sample 2, from Population Y:

- $n_y = 11$
- $\bar{y} = 47,620$
- $s_y = 750$

Use these samples to test $H_0\colon\ \mu_x = \mu_y$ vs $H_1\colon\ \mu_x \neq \mu_y$.

Report:

- The **test statistic**
- The **crictical values** when $\alpha = 0.01$.
- A **decision** when $\alpha = 0.01$.

Solution

- **Test statistic**: $t = 2.944$
  - $s_p = 767.1$
  - $df = 23$
- **Crictical values**: $-2.807, 2.807$
  - Reject if $t > 2.807$
  - Reject if $t < -2.807$
- **Decision**: Reject H_0
