

STAT 400: Homework 05

Fall 2017, UIUC

Due: Friday, October 6, 2:00 PM

Please see the [detailed homework policy document](#) for information about homework formatting, submission, and grading.

Exercise 1

Consider a random variable X with the probability mass function

$$f(x) = \frac{6}{3^x}, \quad x = 2, 3, 4, 5, \dots$$

- (a) Find the moment-generating function of X , $M_X(t)$. Report the function, being sure to indicate the values of t where the function exists.
- (b) Calculate $E[X]$.

Exercise 2

[How much wood would a woodchuck chuck if a woodchuck could chuck wood?](#) Let W denote the amount of wood a woodchuck would chuck per day (in cubic meters) if a woodchuck could chuck wood. Suppose the moment-generating function of W is

$$M_W(t) = 0.1 \cdot e^{3t} + 0.3 \cdot e^{2t} + 0.5 \cdot e^{1t} + 0.1.$$

- (a) Calculate the average amount of wood a woodchuck would chuck per day, $E[W]$.
- (b) Calculate $\text{Var}[W]$.

Exercise 3

Consider a random variable Y with the probability density function

$$f(y) = \frac{|y|}{5}, \quad -1 < y < 3.$$

- (a) Calculate $E[Y]$.
- (b) Calculate $\text{median}[Y]$, the median of Y .

Exercise 4

Suppose that scores on the previous semester's STAT 400 Exam II were not very good. Graphed, their distribution had a shape similar to the probability density function

$$f(s) = \frac{1}{9000}(2s + 10), \quad 40 \leq s \leq 100.$$

Assume that scores on this exam, S , actually follow this distribution. (Note: This distribution does not necessarily reflect reality.)

(a) Suppose 10 students from the class are selected at random. What is the probability that (exactly) 4 of them received a score above 85?

(b) What was the standard deviation of the scores, $\text{SD}[S]$?

(c) What was the class 40th percentile? That is, find a such that $P(S \leq a) = 0.40$.

Exercise 5

Students often worry about the time it takes to complete an exam. Suppose that completion time in hours, T , for the STAT 400 final exam follows a distribution with density

$$f(t) = \frac{2}{27}(t^2 + t), \quad 0 \leq t \leq 3.$$

What is the probability that a randomly chosen student finishes the exam during the second hour of the exam. That is, calculate $P(1 < T < 2)$.