STAT 400
UIUC

Practice Problems #3

Stepanov
Dalpiaz

The following are a number of practice problems that may be *helpful* for completing the homework, and will likely be **very useful** for studying for exams.

1. Prove (show) that

$$\binom{n-1}{r} + \binom{n-1}{r-1} = \binom{n}{r}.$$

(Pascal's equation).

2. A box of candy hearts contains 52 hearts, of which 19 are white, 10 are tan, 7 are pink, 3 are purple, 5 are yellow, 2 are orange, and 6 are green. If you select 9 pieces of candy randomly from the box, without replacement, give the probability that

a) Three of the hearts are white.

b) Three are white, 2 are tan, 1 is pink, 1 is yellow, and 2 are green.

3. Peter takes Computer Science classes, though not to learn, but to meet smart girls. There are 15 other students in the class with Peter, 7 of them are girls. During the semester, students will be working on a project in teams of 4 students. Suppose the students are divided into teams at random.

a) Find the probability that at least 2 out of 3 students on Peter's team are girls.

b) Find the probability that there is at least 1 girl on Peter's team.

c) Find the probability that at most 2 out of 3 students on Peter's team are girls.

4. A small grocery store had 10 cartons of milk, 2 of which were sour.

a) If David is going to buy the sixth carton of milk sold that day at random, compute the probability that he selects a carton of sour milk.

b) If six cartons of milk are sold that day at random, what is the probability that exactly one carton of sour milk is sold.

5. Suppose the number of boxes of Hammermill® paper used by Anytown College Statistics & Probability Department each month is random and has the following probability distribution:

x	$f(x)$
0	0.1
1	0.1
2	0.3
3	0.3
4	0.2

Suppose at the end of each month the department orders the same number of boxes as was used during the month. Suppose each box costs $25. The department has to pay a $5 delivery fee (the delivery fee does not depend on the number of boxes ordered). Then the monthly "paper" bill is $Y = 25 \cdot X + 5$. Find Anytown College Statistics & Probability Department's average monthly "paper" bill and its standard deviation.

6. Suppose we roll a pair of fair 6-sided dice. Let X denote the maximum (the largest) of the outcomes on the two dice. Construct the probability distribution of X and compute its expected value.

7. Consider $f(x) = c(x+1)^2$, $x = 0, 1, 2, 3$.

a) Find c such that $f(x)$ satisfies the conditions of being a p.m.f. for a random variable X.

b) Find the expected value of X.

c) Find the standard deviation of X.

8.

a) Let X be a discrete random variable with p.m.f.

$$f(k) = \frac{c}{a^k}, \quad k = 2, 3, 4, 5, 6, \ldots, \qquad \text{where } c = a(a-1).$$

Recall (Homework #1 Problem 7): this a valid probability distribution.

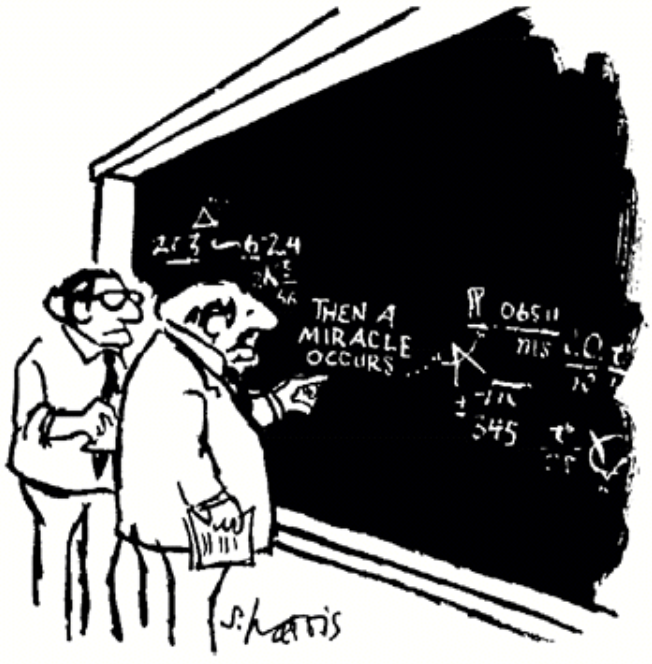
Find $\mu_X = E(X)$.

b) Let X be a discrete random variable with p.m.f.

$$f(k) = c\,\frac{2^k}{k!}, \quad k = 2, 3, 4, 5, 6, \ldots, \qquad \text{where } c = \frac{1}{e^2 - 3}.$$

Recall (Homework #1 Problem 8): this a valid probability distribution.

Find $\mu_X = E(X)$.



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

9 – 10. An oil company believes that the probability of existence of an oil deposit in a certain drilling area is 0.30. Suppose it would cost $100,000 to drill a well. If an oil deposit does exist, the company's profit will be $700,000 (the drilling costs not included). A seismic test that would cost $20,000 is being considered to clarify the likelihood of the presence of oil. The proposed seismic test has the following reliability: when oil does exist in the testing area, the test will indicate so 90% of the time; when oil does not exist in the test area, 20% of the time the test will erroneously indicate that it does exist. There are four possible "states of nature":

θ_1 = an oil deposit does exist and the test result is positive,

θ_2 = an oil deposit does exist, but (and) the test result is negative,

θ_3 = an oil deposit does not exist, but (and) the test result is positive,

θ_4 = an oil deposit does not exist and the test result is negative.

The company can take two possible actions:

a_1 = drill a well without performing the test,

a_2 = perform the test and drill a well only if the test shows presence of oil.

9. a) Find the probabilities of all four states of nature.

That is, find $P(\theta_1)$, $P(\theta_2)$, $P(\theta_3)$, and $P(\theta_4)$.

b) Suppose the test shows presence of oil. What is the probability that an oil deposit does exist?

10. c) Construct the payoff table (profit table) for this problem. That is, find the company's profit for each possible action and each possible state of nature.

	θ_1 Oil +	θ_2 Oil –	θ_3 No Oil +	θ_4 No Oil –
a_1 drill w/o test				
a_2 drill only if +				

d) Find the expected payoff (expected profit, EP) for both actions and determine the optimal action.