STAT 400
UIUC

Practice Problems #8

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SOLUTIONS

The following are a number of practice problems that may be *helpful* for completing the homework, and will likely be **very useful** for studying for exams.

1. a) Let X and Y be random variables with

$$E(X) = \mu_X = 20, \quad SD(X) = \sigma_X = 7,$$
$$E(Y) = \mu_Y = 8, \quad SD(Y) = \sigma_Y = 2, \quad \text{Corr}(X, Y) = \rho = 0.60.$$

Find $E(3X - 5Y)$ and $SD(3X - 5Y)$.

$$E(3X - 5Y) = 3\mu_X - 5\mu_Y = 3 \cdot 20 - 5 \cdot 8 = \mathbf{20}.$$

$$\begin{aligned} \text{Var}(3X - 5Y) &= \text{Cov}(3X - 5Y, 3X - 5Y) \\ &= \text{Cov}(3X, 3X) - \text{Cov}(3X, 5Y) - \text{Cov}(5Y, 3X) + \text{Cov}(5Y, 5Y) \\ &= 9\sigma_X^2 - 30\sigma_{XY} + 25\sigma_Y^2 = 9\sigma_X^2 - 30\rho\sigma_X\sigma_Y + 25\sigma_Y^2 \\ &= 9 \cdot 7^2 - 30 \cdot 0.6 \cdot 7 \cdot 2 + 25 \cdot 2^2 = 289. \end{aligned}$$

$$SD(3X - 5Y) = \sqrt{289} = \mathbf{17}.$$

b) Let X and Y be random variables with

$$SD(X) = \sigma_X = 5, \quad SD(Y) = \sigma_Y = 7, \quad SD(3X + 4Y) = 27.$$

Find $\text{Corr}(X, Y) = \rho$.

$$\begin{aligned} \text{Var}(3X + 4Y) &= \text{Cov}(3X + 4Y, 3X + 4Y) \\ &= \text{Cov}(3X, 3X) + \text{Cov}(3X, 4Y) + \text{Cov}(4Y, 3X) + \text{Cov}(4Y, 4Y) \\ &= 9\sigma_X^2 + 24\sigma_{XY} + 16\sigma_Y^2 = 9\sigma_X^2 + 24\rho\sigma_X\sigma_Y + 16\sigma_Y^2 \\ &= 9 \cdot 5^2 + 24 \cdot \rho \cdot 5 \cdot 7 + 16 \cdot 7^2 = 1009 + 840 \cdot \rho = 729 = 27^2. \end{aligned}$$

$$\Rightarrow \quad \rho = -\frac{\mathbf{1}}{\mathbf{3}}.$$

2. Suppose that you wish to invest in two stocks which both have a current price of \$1. The values of these two stocks in one month are described by two random variables, say, X_1 and X_2. Suppose that the expected values and variances of X_1 and X_2 are μ_1, μ_2, σ_1^2, and σ_2^2, respectively. We also assume that the correlation between the stocks is given by ρ.

Let c denote your initial investment, which is to be invested in the stocks, and assume that shares can be bought up to any percentages. Let w denote the percentage of your investment in stock 1. Finally, let P denote the value of your portfolio (investment) after a month. Then we have that $P = c\,(w X_1 + (1-w) X_2)$, where $0 \le w \le 1$.

a) Find an expression for the expected value of your investment after one month.

The expected value of your investment after one month can be computed as follow

$$\mathbb{E}[P] = c\,(w\mathbb{E}[X_1] + (1-w)\mathbb{E}[X_2]) = c\,(w\mu_1 + (1-w)\mu_2)\,.$$

b) Find an expression for the variance of your investment after one month.

Similarly, we have that

$$\begin{aligned} \mathrm{Var}[P] &= c^2\left(w^2\,\mathrm{Var}[X_1] + (1-w)^2\,\mathrm{Var}[X_2] + 2w(1-w)\,\mathrm{Cov}[X_1, X_2]\right) \\ &= c^2\left(w^2\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho\right) + 2w\left(\sigma_1\sigma_2\rho - \sigma_2^2\right) + \sigma_2^2\right). \end{aligned} \tag{1}$$

c) Find the weights that minimize the risk of your investment.
(Hint: in the classical portfolio theory the risk is simply quantified by the variance.)

First, we consider the first derivative of $\mathrm{Var}[P]$ with respect to w which can easily be obtained using (1). Indeed, we have that

$$\frac{\partial}{\partial w}\,\mathrm{Var}[P] = c^2\left(2w\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho\right) + 2\left(\sigma_1\sigma_2\rho - \sigma_2^2\right)\right).$$

Second, we define $w^\star$ such that

$$\frac{\partial}{\partial w}\,\mathrm{Var}[P]\bigg|_{w=w^\star} = 0.$$

This implies that

$$w^\star\left(\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2\rho\right)=\left(\sigma_2^2-\sigma_1\sigma_2\rho\right)$$

and therefore

$$w^\star=\frac{\sigma_2^2-\sigma_1\sigma_2\rho}{\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2\rho}. \tag{2}$$

Finally, we verify that

$$\frac{\partial^2}{\partial w^2}\operatorname{Var}[P]=2c^2\left(\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2\rho\right)>0$$

since $c>0$ and $\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2\rho=\operatorname{Var}[X_1-X_2]>0$ (assuming of course that either σ_1^2 or σ_2^2 is non-zero). Therefore, $w^\star$, as defined in (2), correspond to the weight minimizing the risk of your investment.

d) Find the correlation which minimizes the risk of the equally weighted portfolio (i.e., $w = 0.5$).

First, we define $P^\star$ as the value of the equally weighted portfolio. The variance $P^\star$ can be obtained directly from (1) and we have that

$$\operatorname{Var}[P^\star]=c^2\left(\frac{1}{4}\left(\sigma_1^2+\sigma_2^2\right)+\frac{1}{2}\sigma_1\sigma_2\rho\right).$$

This function is linear in ρ and can therefore be written as

$$f(\rho)=\operatorname{Var}[P^\star]=a+b\rho$$

where both constants a and b are positive (assuming again that either σ_1^2 or σ_2^2 is non-zero). This implies that the variance of $P^\star$ is minimized for $\rho=-1$.

3. In Anytown, the price of a gallon of milk (X) varies from day to day according to normal distribution with mean \$3.00 and standard deviation \$0.20. The price of a package of Oreo cookies (Y) also varies from day to day according to normal distribution with mean \$2.70 and standard deviation \$0.15. Assume the prices of a gallon of milk and a package of Oreo cookies are independent.

a) Find the probability that on a given day, the price of a package of Oreo cookies is higher than the price of a gallon of milk. That is, find $P(Y > X)$.

$P(Y > X) = P(X - Y < 0)$.

$X - Y$ has Normal distribution with mean $E(X - Y) = 3.00 - 2.70 = \$0.30$
and variance $\operatorname{Var}(X - Y) = \operatorname{Var}(X) + \operatorname{Var}(Y) = 0.20^2 + 0.15^2 = 0.0625$
(standard deviation $= \$0.25$).

$$P(X - Y < 0) = P\left(Z < \frac{0 - 0.30}{0.25}\right) = P(Z < -1.20) = \Phi(-1.20) = \mathbf{0.1151}.$$

b) Alex is planning a Milk-and-Oreos party for his imaginary friends. He buys 4 gallons of milk and 7 packages of Oreo cookies. Find the probability that he paid less than \$30. That is, find $P(4X + 7Y < 30)$.

$4X + 7Y$ has Normal distribution with mean $E(4X + 7Y) = 4 \cdot 3.00 + 7 \cdot 2.70 = \$30.90$
and variance $\operatorname{Var}(4X + 7Y) = 16 \cdot \operatorname{Var}(X) + 49 \cdot \operatorname{Var}(Y) = 16 \cdot 0.20^2 + 49 \cdot 0.15^2 = 1.7425$
(standard deviation $\approx \$1.32$).

$$P(4X + 7Y < 30) = P\left(Z < \frac{30 - 30.90}{1.32}\right) = P(Z < -0.68) = \Phi(-0.68) = \mathbf{0.2483}.$$

4. In post-apocalyptic Neverland, clean water is sold in "2-liter" bottles off the back of a truck that arrives every day. The arrival time is normally distributed with mean 3:06 pm and standard deviation 24 minutes. The time it takes for the water to sell out is also normally distributed with mean 30 minutes and standard deviation 7 minutes. The price of a "2-liter" bottle fluctuates from day to day according to a Normal distribution with mean 123 rubles and standard deviation 4 rubles. The amount of water in a "2-liter" bottle varies from bottle to bottle according to a Normal distribution with mean 65 ounces and standard deviation 1.6 ounces. Assume that all days, all times, and all bottles are independent.

a) On April 3, 2020, you buy four "2-liter" bottles of clean water. What is the probability that it costs you more than 500 rubles?

Price has a Normal distribution with mean $\mu_{\text{Price}} = 123$ rubles
and standard deviation $\sigma_{\text{Price}} = 4$ rubles.

$4 \times$ Price has a Normal distribution with mean $4 \times \mu_{\text{Price}} = 4 \times 123 = 492$ rubles
and variance $4^2 \times (\sigma_{\text{Price}})^2 = 4^2 \times 4^2 = 256$ ruble^2
(standard deviation 16 rubles).

$$P(4 \times \text{Price} > 500) = P\left(Z > \frac{500 - 492}{16}\right) = P(Z > 0.50) = \mathbf{0.3085}.$$

b) On April 3, 2020, you buy four "2-liter" bottles of clean water. What is the probability that you get more than 2 gallons (256 ounces) of clean water?

W_1, W_2, W_3, W_4 are independent normally distributed random variables with
mean $\mu_W = 65$ ounces and standard deviation $\sigma_W = 1.6$ ounces.

$E(W_1 + W_2 + W_3 + W_4) = 65 + 65 + 65 + 65 = 260$ ounces.

$\text{Var}(W_1 + W_2 + W_3 + W_4) = 1.6^2 + 1.6^2 + 1.6^2 + 1.6^2 = 10.24$ ounce^2,
$\text{SD}(W_1 + W_2 + W_3 + W_4) = 3.2$ ounces.

$$P(W_1 + W_2 + W_3 + W_4 > 256) \;=\; P\left(Z > \frac{256 - 260}{3.2}\right) \;=\; P(Z > -1.25) \;=\; \mathbf{0.8944}.$$

c) On April 3, 2020, you arrive to the lot where the truck stops at exactly 3:00 pm. What is the probability that the water is sold out by the time you arrive?

T_A has a Normal distribution with mean $\mu_A = 3{:}06$ and standard deviation $\sigma_A = 24$ min
T_S has a Normal distribution with mean $\mu_S = 30$ min and standard deviation $\sigma_S = 7$ min

$E(T_A + T_S) \;=\; 3{:}06 + 0{:}30 \;=\; 3{:}36.$

$Var(T_A + T_S) \;=\; 24^2 + 7^2 \;=\; 625 \text{ min}^2,$ $\qquad SD(T_A + T_S) \;=\; 25$ min.

$$P(T_A + T_S < 3{:}00) \;=\; P\left(Z < \frac{0 - 36}{25}\right) \;=\; P(Z < -1.44) \;=\; \mathbf{0.0749}.$$

d) What is the probability that the truck arrives before 3:00 pm on exactly 3 days in one week?

$$P(T_A < 3{:}00) \;=\; P\left(Z < \frac{0 - 6}{24}\right) \;=\; P(Z < -0.25) \;=\; 0.4013.$$

Let X = the number of days in one week when the truck arrives before 3:00 pm.
Then X has a Binomial distribution, $n = 7$, $p = 0.4013$.

$$P(X = 3) \;=\; {}_7C_3 \times 0.4013^3 \times 0.5987^4 \;=\; \mathbf{0.2906}.$$

5. At Sam's Butcher Shop, ground beef packages vary in weight according to a normal distribution with a mean of 3.1 pounds and a standard deviation of 0.2 pounds, and are sold for $1 per pound. Packages of Bratwurst vary in weight according to a normal distribution with a mean of 2.6 pounds and a standard deviation of 0.3 pounds, and are sold for $3 per pound. Anticipating nice weather during the weekend, Dick buys two packages of ground beef and one package of Bratwurst, selecting the packages at random. What is the probability that Dick would exceed the $15 limit "suggested" by his wife Jane? (Assume independence.)

$$\text{Total} = \$3 \cdot \text{Bratwurst} + \$1 \cdot \text{GrBeef}_1 + \$1 \cdot \text{GrBeef}_2.$$

$$\text{E(Total)} = \$3 \cdot \text{E(Bratwurst)} + \$1 \cdot \text{E}(\text{GrBeef}_1) + \$1 \cdot \text{E}(\text{GrBeef}_2)$$

$$= \$3 \cdot 2.6 + \$1 \cdot 3.1 + \$1 \cdot 3.1 = \$14.$$

$$\text{Var(Total)} = 3^2 \cdot \text{Var(Bratwurst)} + 1^2 \cdot \text{Var}(\text{GrBeef}_1) + 1^2 \cdot \text{Var}(\text{GrBeef}_2)$$

$$= 3^2 \cdot 0.3^2 + 1^2 \cdot 0.2^2 + 1^2 \cdot 0.2^2 = 0.89.$$

$\text{SD(Total)} = \sqrt{0.89} = \$0.9434.$ Total has Normal distribution.

$$P(\text{ Total} > 15) = P\left(Z > \frac{15-14}{0.9434} \right) = P(Z > 1.06) = 1 - 0.8554 = \mathbf{0.1446}.$$

6. The distribution of the baggage weights for passengers using a particular airline has a mean of 20 lbs and a standard deviation of 5 lbs. What is the probability that for (a random sample of) 100 passengers …

a) the total luggage weight is less than 2,100 lbs?

$\mathrm{E}(\text{Total}) = 100 \times 20 = 2{,}000.$

$\mathrm{Var}(\text{Total}) = 100 \times 5^2 = 2{,}500.$ $\qquad$ $\mathrm{SD}(\text{Total}) = 50.$

$n = 100$ – large. Central Limit Theorem:

Total is approximately normally distributed.

$$\mathrm{P}(\text{Total} < 2{,}100) \approx \mathrm{P}\left(Z < \frac{2{,}100 - 2{,}000}{50}\right) = \mathrm{P}(Z < 2.00) = \mathbf{0.9772}.$$

OR

Total < 2,100 $\Rightarrow$ Average $< \frac{2{,}100}{100} = 21.$ Need $\mathrm{P}(\overline{X} < 21) = ?$

$n = 100$ – large. Central Limit Theorem: $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \approx Z.$

$$\mathrm{P}(\overline{X} < 21) \approx \mathrm{P}\left(Z < \frac{21 - 20}{5/\sqrt{100}}\right) = \mathrm{P}(Z < 2.00) = \mathbf{0.9772}.$$

b) the sample mean weight is within 0.5 lb of the overall mean? That is, what is the probability that the sample mean weight is between 19.5 and 20.5 lbs?

$$\mathrm{P}(19.5 < \overline{X} < 20.5) \approx \mathrm{P}\left(\frac{19.5 - 20}{5/\sqrt{100}} < Z < \frac{20.5 - 20}{5/\sqrt{100}}\right) = \mathrm{P}(-1.00 < Z < 1.00)$$

$$= 0.8413 - 0.1587 = \mathbf{0.6826}.$$

7. The amount of cereal dispensed into "16-ounce" boxes of *Captain Crisp* cereal was normally distributed with mean 16.12 ounces and standard deviation 0.20 ounces.

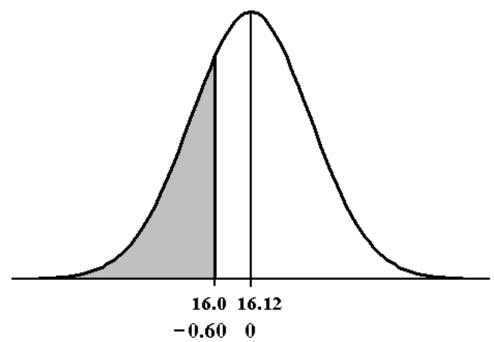
a) What proportion of boxes are "underfilled"? That is, what is the probability that the amount dispensed into a box is less than 16 ounces?

$\mu = 16.12$, $\sigma = 0.20$.

$$P(X < 16.0) = P\left(Z < \frac{16.0 - 16.12}{0.20}\right)$$

$$= P(Z < -0.60)$$

$$= \mathbf{0.2743}.$$



b) Find the probability that exactly 2 out of 9 randomly and independently selected boxes of cereal contain less than 16 ounces.

Let Y = number of boxes of cereal (out of 9) that contain less than 16 ounces.

Then Y has Binomial distribution, $n = 9$, $p = 0.2743$ (see part (a)).

Need $P(Y = 2) = ?$ $P(Y = k) = {}_nC_k \cdot p^k \cdot (1-p)^{n-k}$

$$P(Y = 2) = {}_9C_2 \cdot (0.2743)^2 \cdot (0.7257)^7 = \mathbf{0.2871}.$$

c) Find the probability that the sample mean amount of cereal for a random sample of 9 boxes is less than 16 ounces.

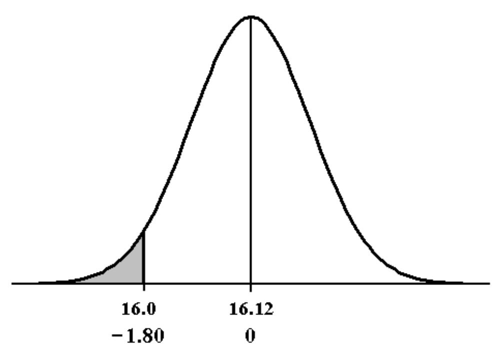
$n = 9$. Need $P(\overline{X} \le 16.0) = ?$

We sample from a normally distributed population. $\Rightarrow$ $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} = Z.$

$$P(\overline{X} \le 16.0) = P\left(Z \le \frac{16.0 - 16.12}{0.20 / \sqrt{9}}\right)$$

$$= P(Z \le -1.80)$$

$$= \mathbf{0.0359}.$$



d) Suppose that the machine can be adjusted to change the mean while the standard deviation remains at 0.20 ounces. What must the mean be so that only 20% of all the boxes are "underfilled"?

$\sigma = 0.20.$ Want $P(X < 16.0) = 0.20.$ $\mu = ?$

Find z such that $P(Z < z) = 0.20.$

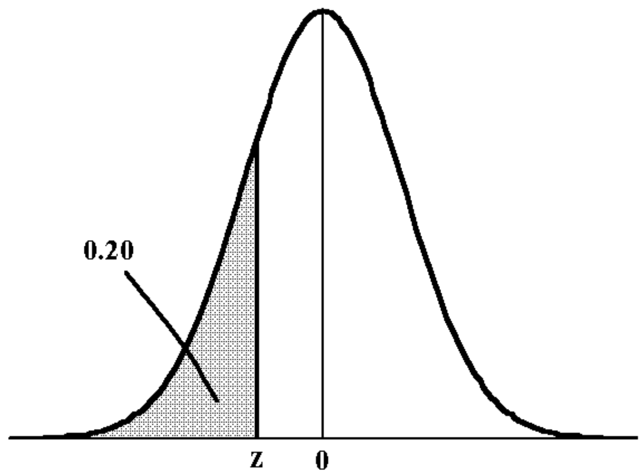
The area to the left of z is 0.20.
Using the standard normal table,

$z = -0.84.$

$x = \mu + \sigma \cdot z.$

$16.0 = \mu + 0.20 \cdot (-0.84).$

$\mu =$ **16.168**.



8. The weight of an almond varies with mean 0.051 ounce and standard deviation 0.018 ounce.

a) What is the probability (approximately) that the total weight (of a random sample) of 60 almonds is less than 3 ounces?

Total < 3 $\Rightarrow$ Average < $\frac{3}{60}$ = 0.05. Need $P(\overline{X} < 0.05) = ?$

n = 60 – large. Central Limit Theorem: $\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}} \approx Z$.

$$P(\overline{X} < 0.05) \approx P\left(Z < \frac{0.05-0.051}{0.018/\sqrt{60}}\right) = P(Z < -0.43) = \Phi(-0.43) = \mathbf{0.3336}.$$

OR

$E(\text{Total}) = 60 \times 0.051 = 3.06.$

$Var(\text{Total}) = 60 \times 0.018^2 = 0.01944.$ $SD(\text{Total}) \approx 0.1394.$

n = 60 – large. Total is approximately normally distributed.

$$P(\text{Total} < 3) \approx P\left(Z < \frac{3-3.06}{0.1394}\right) = P(Z < -0.43) = \Phi(-0.43) = \mathbf{0.3336}.$$

b) Determine the sample size (the number of almonds) needed to have the probability of at least 0.80 that the total weight is greater than 16 ounces.

$P(Z > -0.84) = 0.7995 \approx 0.80.$ $\dfrac{\text{Total} - n\cdot\mu}{\sqrt{n}\cdot\sigma} \approx Z.$

$\dfrac{16 - n\cdot 0.051}{\sqrt{n}\cdot 0.018} = -0.84.$ $0.051\cdot n - 0.84\cdot 0.018\cdot\sqrt{n} - 16 = 0.$

$$\Rightarrow \quad \sqrt{n} = \frac{0.84\cdot 0.018 \pm \sqrt{0.84^2\cdot 0.018^2 + 4\cdot 0.051\cdot 16}}{2\cdot 0.051} \approx 17.861 \text{ or } -17.565$$

$\Rightarrow$ n = **at least 319**. (it actually should be at least 320 [rounding])

9. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $\text{Gamma}(\alpha, \theta)$ distribution. That is,

$$f(x;\alpha,\theta) = \frac{1}{\Gamma(\alpha)\,\theta^{\alpha}}\, x^{\alpha-1} e^{-x/\theta}, \qquad 0 < x < \infty, \qquad\qquad \alpha > 0,\ \theta > 0.$$

Suppose α is known.

a) Obtain a method of moments estimator of θ, $\tilde{\theta}$.

$$\mu = \alpha\,\theta. \qquad\qquad \overline{X} = \alpha\,\tilde{\theta}. \qquad\qquad \tilde{\theta} = \frac{\overline{X}}{\alpha}.$$

b) Obtain the maximum likelihood estimator of θ, $\hat{\theta}$.

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \left(\frac{1}{\Gamma(\alpha)\,\theta^{\alpha}}\right)^{n} \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} \exp\left\{-\frac{1}{\theta}\sum_{i=1}^{n} x_i\right\}.$$

$$\ln L(\theta) = -n \ln\Gamma(\alpha) - n\,\alpha \ln\theta + (\alpha-1)\sum_{i=1}^{n} \ln x_i - \frac{1}{\theta}\sum_{i=1}^{n} x_i\,.$$

$$\frac{d}{d\theta}\ln L(\theta) = -\frac{n\alpha}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n} x_i = 0. \qquad\qquad \hat{\theta} = \frac{1}{n\alpha}\sum_{i=1}^{n} X_i = \frac{\overline{X}}{\alpha}.$$

For fun:

c) Is $\hat{\theta}$ an unbiased estimator for θ? *Justify your answer.*

"Hint": $E(\overline{X}) = \mu$.

$$E(\hat{\theta}) = E\left(\frac{\overline{X}}{\alpha}\right) = \frac{\mu}{\alpha} = \frac{\alpha\theta}{\alpha} = \theta.$$

$\hat{\theta}$ is an unbiased estimator for θ.

d) Find $\operatorname{Var}(\hat{\theta})$.

"Hint": $\operatorname{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$.

$$\operatorname{Var}(\hat{\theta}) = \operatorname{Var}\left(\frac{\overline{X}}{\alpha}\right) = \frac{1}{\alpha^2}\operatorname{Var}(\overline{X}) \quad \frac{1}{\alpha^2}\cdot\frac{\sigma^2}{n} = \frac{\alpha\theta^2}{\alpha^2 n} = \frac{\theta^2}{\alpha n}.$$

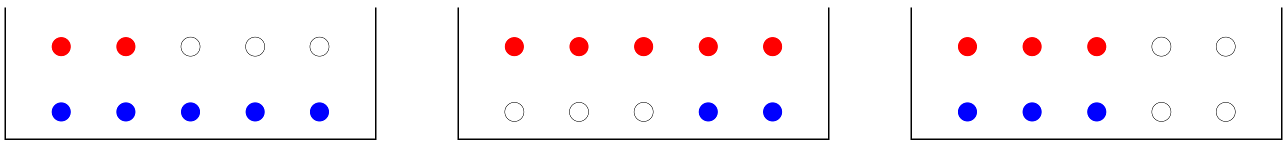
e) Find $\operatorname{MSE}(\hat{\theta})$.

$$\operatorname{bias}(\hat{\theta}) = E(\hat{\theta}) - \theta = 0 \quad \text{and} \quad \operatorname{Var}(\hat{\theta}) = \frac{\theta^2}{\alpha n}.$$

$$\Rightarrow \qquad \operatorname{MSE}(\hat{\theta}) = E\left[(\hat{\theta}-\theta)^2\right] = (\operatorname{bias}(\hat{\theta}))^2 + \operatorname{Var}(\hat{\theta}) = 0 + \frac{\theta^2}{\alpha n} = \frac{\theta^2}{\alpha n}.$$

Note that $\operatorname{MSE}(\hat{\theta}) = \dfrac{\theta^2}{\alpha n} \to 0$ as $n \to \infty$.

10. Imagine you are selected as a contestant on *The Price is Right*. The host, Bob Barker, shows you three boxes with marbles in them. Box #1 contains 2 red, 3 white and 5 blue marbles. Box #2 contains 5 red, 3 white and 2 blue marbles. Lastly, Box #3 contains 3 red, 4 white and 3 blue marbles. One of Bob's assistants will pick marbles from one of the boxes after they are hidden behind a curtain. If you correctly guess which box the marbles were picked from, you win a brand new car! Which box (or boxes) would you pick if …



Box #1 Box #2 Box #3

a) one white and one blue marble are selected, with replacement?

Box #1 $$P(W\,B) + P(B\,W) = \frac{3}{10}\times\frac{5}{10}+\frac{5}{10}\times\frac{3}{10} = 0.30 \quad \leftarrow \text{largest}$$

Box #2 $$P(W\,B) + P(B\,W) = \frac{3}{10}\times\frac{2}{10}+\frac{2}{10}\times\frac{3}{10} = 0.12$$

Box #3 $$P(W\,B) + P(B\,W) = \frac{4}{10}\times\frac{3}{10}+\frac{3}{10}\times\frac{4}{10} = 0.24$$

Box #1.

b) one red and one blue marble are selected, without replacement?

Box #1 $$P(R\,B) + P(B\,R) = \frac{2}{10}\times\frac{5}{9}+\frac{5}{10}\times\frac{2}{9} = \frac{2}{9} \quad \leftarrow \text{largest}$$

Box #2 $$P(R\,B) + P(B\,R) = \frac{5}{10}\times\frac{2}{9}+\frac{2}{10}\times\frac{5}{9} = \frac{2}{9} \quad \leftarrow \text{largest}$$

Box #3 $$P(R\,B) + P(B\,R) = \frac{3}{10}\times\frac{3}{9}+\frac{3}{10}\times\frac{3}{9} = \frac{2}{10}$$

Box #1 or **Box #2**.

OR

Box #1 $\dfrac{{}_2C_1 \times {}_3C_0 \times {}_5C_1}{{}_{10}C_2} = \dfrac{2}{9}$ ← largest

Box #2 $\dfrac{{}_5C_1 \times {}_3C_0 \times {}_2C_1}{{}_{10}C_2} = \dfrac{2}{9}$ ← largest

Box #3 $\dfrac{{}_3C_1 \times {}_4C_0 \times {}_3C_1}{{}_{10}C_2} = \dfrac{2}{10}$

Box #1 or **Box #2**.

c) one red, one white, and one blue marble are selected, without replacement?

Box #1 $\dfrac{{}_2C_1 \times {}_3C_1 \times {}_5C_1}{{}_{10}C_3} = 0.25$

Box #2 $\dfrac{{}_5C_1 \times {}_3C_1 \times {}_2C_1}{{}_{10}C_3} = 0.25$

Box #3 $\dfrac{{}_3C_1 \times {}_4C_1 \times {}_3C_1}{{}_{10}C_3} = 0.30$ ← largest

Box #3.