

## Practice Problems #12

The following are a number of practice problems that may be *helpful* for completing the homework, and will likely be **very useful** for studying for exams.

1. The April 18, 1994 issue of *Time* magazine reported the results of a telephone poll of 800 adult Americans, 605 nonsmokers, who were asked the following question: “Should the federal tax on cigarettes be raised by \$1.25 to pay for health care reform?” Let  $p_1$  and  $p_2$  equal the proportions of nonsmokers and smokers, respectively, who say yes to this question.  $y_1 = 351$  nonsmokers and  $y_2 = 41$  smokers said yes.
  - a) With  $\alpha = 0.05$ , test  $H_0: p_1 = p_2$  vs.  $H_1: p_1 \neq p_2$ .
  - b) Find a 95% confidence interval for  $p_1 - p_2$ .
  - c) Find a 95% confidence interval for  $p$ , the proportion of adult Americans who would say yes.
  
2. A new method of storing snap beans is believed to retain more ascorbic acid than the old method. In an experiment, snap beans were harvested under uniform conditions and frozen in 25 equal-size packages. Ten of those packages were randomly selected and stored according to the new method, and the other 15 packages were stored by the old method. Subsequently, ascorbic acid determinations (in mg/kg) were made, and the following summary statistics were calculated.

	New Method	Old Method
(sample) mean ascorbic acid	435	410
(sample) standard deviation	20	45

- a) Use Welch’s T to construct a 95% confidence interval for  $\mu_{\text{New}} - \mu_{\text{Old}}$ .
- b) Test  $H_0: \mu_{\text{New}} = \mu_{\text{Old}}$  vs.  $H_1: \mu_{\text{New}} > \mu_{\text{Old}}$  at a 5% level of significance.

3. Assume that the distributions of  $X$  and  $Y$  are  $N(\mu_1, \sigma^2)$  and  $N(\mu_2, \sigma^2)$ , respectively. Given the  $n = 4$  observations of  $X$ ,

105, 130, 135, 150

and the  $m = 6$  observations of  $Y$ ,

126, 141, 146, 156, 166, 171

find the p-value (approximately) for the test  $H_0: \mu_1 = \mu_2$  vs.  $H_1: \mu_1 \neq \mu_2$ .

“Hint”: Assume the population variances are equal.

4. A random sample of 9 adult elephants had the sample mean weight of 12,240 pounds and the sample standard deviation of 450 pounds. A random sample of 16 adult hippos had the sample mean weight of 5,700 pounds and the sample standard deviation of 400 pounds. Assume that the two populations are approximately normally distributed, Construct a 95% confidence interval for the difference between their overall average weights of adult elephants and adult hippos.

- a) Assume that the overall standard deviations are equal.
- b) Do NOT assume that the overall standard deviations are equal. Use Welch’s T.

5. Six children are tested for pulse rate before and after watching a violent movie with the following results.

<u>Child</u>	<u>Before</u>	<u>After</u>
1	102	112
2	96	108
3	89	94
4	104	112
5	90	102
6	85	98

Using the paired  $t$  test, test for differences in the before and after mean pulse rates.

Use  $\alpha = 0.05$  and use a two-sided test.

6. Crosses of mice will produce either gray, brown, or albino offspring. Mendel's model predicts that the probability of a gray offspring is  $9/16$ ; the probability of a brown offspring is  $3/16$ ; and the probability of an albino offspring is  $4/16$ .

a) An experiment to assess the validity of Mendel's theory produces the following data: 35 gray offspring; 20 brown offspring; and 25 albino offspring. Test

$$H_0: p_1 = 9/16, p_2 = 3/16, p_3 = 4/16 \quad \text{vs} \quad H_1: \text{not } H_0$$

i) at  $\alpha = 0.05$ ,                      ii) at  $\alpha = 0.10$ .

b) Suppose the experiment in part (a) is repeated, but with twice as many observations. Suppose also that we happened to get the same proportions, namely, 70 gray offspring; 40 brown offspring; and 50 albino offspring. Repeat part (a) in this case, using  $\alpha = 0.01$ .

7. Suppose we toss a 6-sided die 120 times and count how many times each outcome (1 through 6) occurs. We obtain the following results:

Outcome	1	2	3	4	5	6
Observed frequency	14	24	28	17	12	25

We want to use the chi-square goodness-of-fit test to test the hypothesis that the die is fair (balanced) using a 5% level of significance.

- State the null hypothesis.
- Calculate the values of the chi-square test statistic.
- Find the critical value  $\chi^2_{\alpha}$ .
- Test the hypothesis that the die is fair (balanced) using a 5% level of significance.

8. An article in *Business Week* reports profits and losses of firms by industry. A random sample of 100 firms is selected, and for each firm in the sample, we record whether the company made money or lost money, and whether or not the firm is a service company.

The data are summarized in the  $2 \times 2$  table below. Use a 10% level of significance to test whether the two events “the company made profit this year” and “the company is in the service industry” are independent.

	Industry Type	
	Service	Nonservice
Profit	32	38
Loss	8	22

9. A group of 250 children (125 boys and 125 girls) were asked to identify their favorite color. We obtain the following data:

Sex	Favorite Color				
	Red	Green	Blue	Pink	Purple
Boys	35	25	35	10	20
Girls	25	20	25	30	25

A toy manufacturer wants to know if the color preferences of boys and girls differ. Perform  $\chi^2$  test of homogeneity using a 1% level of significance.

10. A breakfast cereal manufacturer wants to know whether individual preferences for types of breakfast cereal sweetener are associated with the age of the buyer. In a survey, sweetener preferences were matched to the consumers’ age group. From a random sample of 500 responses, the results were as follows:

Cereal	Age (Years)			
	15 – 25	26 – 40	41 – 60	Over 60
Sugar Sweetened	50	60	55	35
Fruit Sweetened	25	35	55	35
Natural	25	55	40	30

Test whether sweetener preferences and age are independent at a 1% level of significance.