Week 01 Discussion

1. Consider the following experiment:

A letter is chosen at random from the word **STATISTICS**.

a) List all possible outcomes and their probabilities.

b) What is the probability that the letter selected is a vowel?

2. **1.1-4** **1.2-4** 1.2-2

A fair coin is tossed four times, and the sequence of heads and tails is observed.

a) List each of the 16 sequences in the sample space S.

b) Let events A, B, C, and D be given by A = { at least 3 heads }, B = { at most 2 heads }, C = { heads on the third toss }, and D = { 1 head and 3 tails }. If the probability set function assigns 1/16 to each outcome in the sample space, find

(i) $P(A)$, (ii) $P(A \cap B)$, (iii) $P(B)$,

(iv) $P(A \cap C)$, (v) $P(D)$, (vi) $P(A \cup C)$

(vii) $P(B \cap D)$.

3. **1.1-6** **1.2-8** 1.2-6

If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find …

a) $P(A \cup B)$; b) $P(A \cap B')$; c) $P(A' \cup B')$.

4. Suppose that $P(A) = 0.40$, $P(B) = 0.50$, $P(A \cup B) = 0.70$. Find …

a) $P(A \cap B)$; b) $P(A' \cap B')$; c) $P(A' \cup B')$.

5. **1.2-16** 1.2-14

The five numbers 1, 2, 3, 4, and 5 are written respectively on five disks of the same size and placed in a hat. Two disks are drawn without replacement from the hat, and the numbers written on them are observed.

a) List the 10 possible outcomes of this experiment as unordered pairs of numbers.

b) If each of the 10 outcomes has probability 1/10, assign a value to the probability that the sum of the two numbers drawn is

(i) 3; (ii) between 6 and 8 inclusive.

6. Suppose that $P(A) = 0.60$ and $P(B) = 0.50$.

a) Can A and B be mutually exclusive? Why or why not? What is the minimum possible value of $P(A \cap B)$? What is the maximum possible value of $P(A \cap B)$?

b) What is the minimum possible value of $P(A \cup B)$? What is the maximum possible value of $P(A \cup B)$?

7. Suppose that $P(A) = 0.40$ and $P(B) = 0.30$.

a) Can A and B be mutually exclusive? Why or why not? What is the minimum possible value of $P(A \cap B)$? What is the maximum possible value of $P(A \cap B)$?

b) What is the minimum possible value of $P(A \cup B)$? What is the maximum possible value of $P(A \cup B)$?

8. Suppose $P(A) = 0.40$, $P(B) = 0.34$, $P(C) = 0.55$,
$P(A \cap B) = 0.19$, $P(A \cap C) = 0.25$, $P(B \cap C) = 0.17$,
$P(A \cap B \cap C) = 0.07$.

a) Find $P(A \cup B \cup C)$. b) Find $P((A \cap B) \cup C)$.

c) Find $P(A \cap (B \cup C))$.

Answers:

1. Consider the following experiment:

A letter is chosen at random from the word **STATISTICS**.

a) List all possible outcomes and their probabilities.

10 letters: 1 **A**, 1 **C**, 2 **I**, 3 **S**, 3 **T**.

Possible Outcomes:	**A**,	**C**,	**I**,	**S**,	**T**
Probabilities:	0.10	0.10	0.20	0.30	0.30

b) What is the probability that the letter selected is a vowel?

P(vowel) = P(**A**) + P(**I**) = 0.10 + 0.20 = **0.30**.

2. **1.1-4** **1.2-4** 1.2-2

A fair coin is tossed four times, and the sequence of heads and tails is observed.

a) List each of the 16 sequences in the sample space *S*.

H H H H	H T H H	T H H H	T T H H
H H H T	H T H T	T H H T	T T H T
H H T H	H T T H	T H T H	T T T H
H H T T	H T T T	T H T T	T T T T

b) Let events A, B, C, and D be given by A = { at least 3 heads }, B = { at most 2 heads }, C = { heads on the third toss }, and D = { 1 head and 3 tails }. If the probability set function assigns 1/16 to each outcome in the sample space, find

(i) $P(A)$, (ii) $P(A \cap B)$, (iii) $P(B)$,

(iv) $P(A \cap C)$, (v) $P(D)$, (vi) $P(A \cup C)$

(vii) $P(B \cap D)$.

A	B	C	D
H H H H	H H T T	H H H H	H T T T
H H H T	H T H T	H H H T	T H T T
H H T H	H T T H	H T H H	T T H T
H T H H	H T T T	H T H T	T T T H
T H H H	T H H T	T H H H	
	T H T H	T H H T	
	T H T T	T T H H	
	T T H H	T T H T	
	T T H T		
	T T T H		
	T T T T		

(i) $P(A) = \mathbf{5/16}$

(ii) $A \cap B = \varnothing$ $P(A \cap B) = \mathbf{0}$

(iii) $P(B) = \mathbf{11/16}$

(iv) $A \cap C = \{ HHHH, HHHT, HTHH, THHH \}$

$P(A \cap C) = \mathbf{4/16}$

(v) $P(D) = \mathbf{4/16}$

(vi) $P(A \cup C) = P(A) + P(C) - P(A \cap C) = 5/16 + 8/16 - 4/16 = \mathbf{9/16}$

(vii) $D \subset B$ $B \cap D = D$ $P(B \cap D) = P(D) = \mathbf{4/16}$

3. **1.1-6** **1.2-8** 1.2-6

If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find …

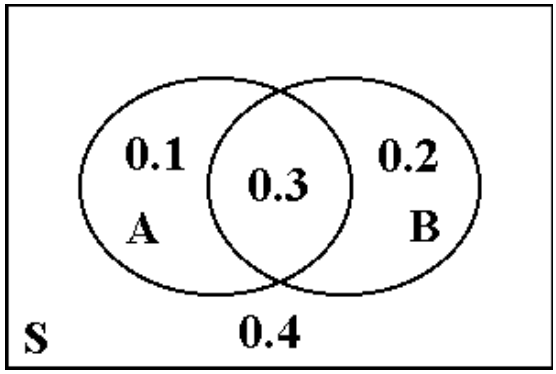
a) $P(A \cup B)$; b) $P(A \cap B')$; c) $P(A' \cup B')$.

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.3 = \mathbf{0.6}$.

b) $P(A \cap B') = P(A) - P(A \cap B) = 0.4 - 0.3 = \mathbf{0.1}$.

c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.3 = \mathbf{0.7}$.

OR



a) $P(A \cup B) = \mathbf{0.6}$.

b) $P(A \cap B') = \mathbf{0.1}$.

c) $P(A' \cup B') = \mathbf{0.7}$.

4. Suppose that $P(A) = 0.40$, $P(B) = 0.50$, $P(A \cup B) = 0.70$. Find …

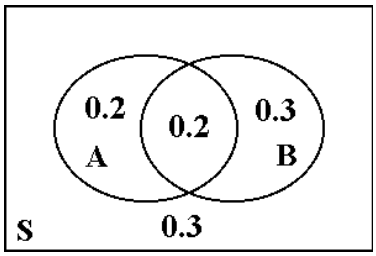
a) $P(A \cap B)$; b) $P(A' \cap B')$; c) $P(A' \cup B')$.

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.4 + 0.5 - P(A \cap B)$ $P(A \cap B) = \mathbf{0.2}$.

b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.7 = \mathbf{0.3}$.

c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.2 = \mathbf{0.8}$.



5. **1.2-16** 1.2-14

The five numbers 1, 2, 3, 4, and 5 are written respectively on five disks of the same size and placed in a hat. Two disks are drawn without replacement from the hat, and the numbers written on them are observed.

a) List the 10 possible outcomes of this experiment as unordered pairs of numbers.

$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(2,3)$
$(2,4)$	$(2,5)$	$(3,4)$	$(3,5)$	$(4,5)$

b) If each of the 10 outcomes has probability 1/10, assign a value to the probability that the sum of the two numbers drawn is

(i) 3; (ii) between 6 and 8 inclusive.

(i) $\frac{1}{10}$ (ii) $\frac{5}{10}$

6. Suppose that $P(A) = 0.60$ and $P(B) = 0.50$.

a) Can A and B be mutually exclusive? Why or why not? What is the minimum possible value of $P(A \cap B)$? What is the maximum possible value of $P(A \cap B)$?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

If A and B are mutually exclusive, $P(A \cap B) = 0$.

No, A and B cannot be mutually exclusive since P(A) + P(B) = 1.1 > 1.

[Minimum possible value of $P(A \cap B)$] = **0.10**.

[Maximum possible value of $P(A \cap B)$] = **0.50** (the smaller of P(A) and P(B)).

b) What is the minimum possible value of $P(A \cup B)$? What is the maximum possible value of $P(A \cup B)$?

[Minimum possible value of $P(A \cup B)$] = **0.60** (the larger of P(A) and P(B)).

[Maximum possible value of $P(A \cup B)$] = **1**.

7. Suppose that $P(A) = 0.40$ and $P(B) = 0.30$.

a) Can A and B be mutually exclusive? Why or why not? What is the minimum possible value of $P(A \cap B)$? What is the maximum possible value of $P(A \cap B)$?

Yes, A and B can be mutually exclusive since $P(A) + P(B) = 0.7 \leq 1$.

[Minimum possible value of $P(A \cap B)$] = **0**.

[Maximum possible value of $P(A \cap B)$] = **0.30** (the smaller of P(A) and P(B)).

b) What is the minimum possible value of $P(A \cup B)$? What is the maximum possible value of $P(A \cup B)$?

[Minimum possible value of $P(A \cup B)$] = **0.40** (the larger of P(A) and P(B)).

[Maximum possible value of $P(A \cup B)$] = **0.70**.

8. Suppose

$P(A) = 0.40$,

$P(B) = 0.34$,

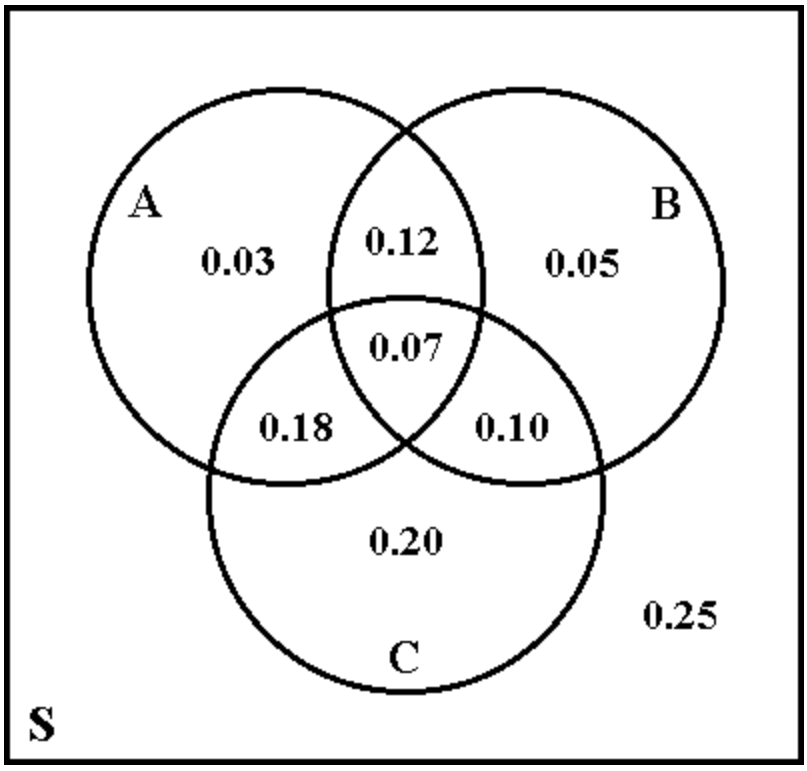
$P(C) = 0.55$,

$P(A \cap B) = 0.19$,

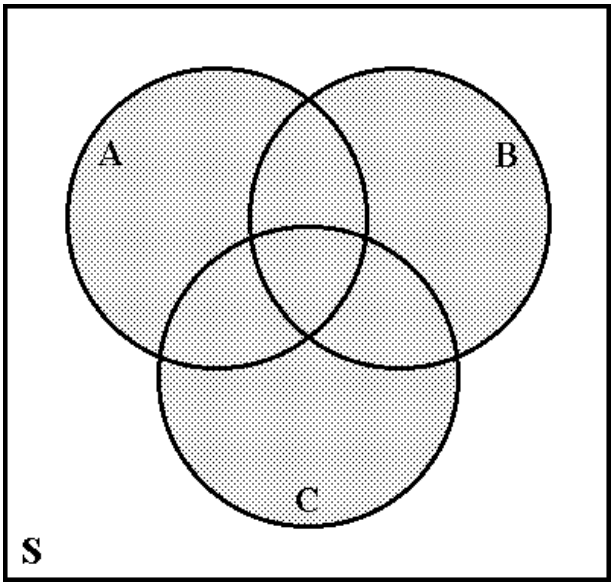
$P(A \cap C) = 0.25$,

$P(B \cap C) = 0.17$,

$P(A \cap B \cap C) = 0.07$.

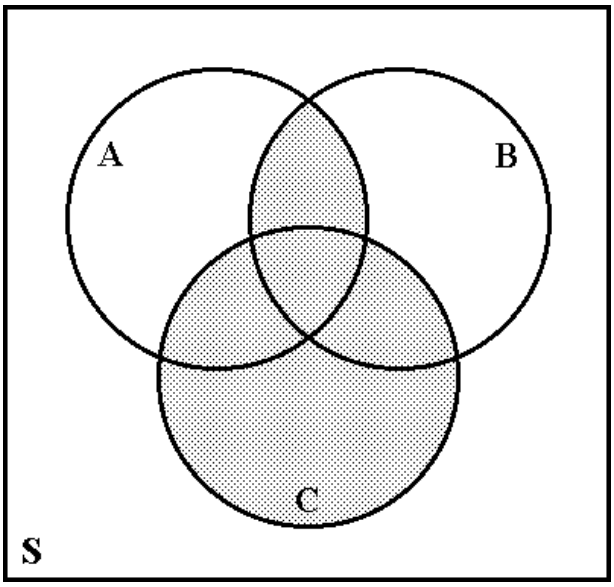


a) Find $P(A \cup B \cup C)$.



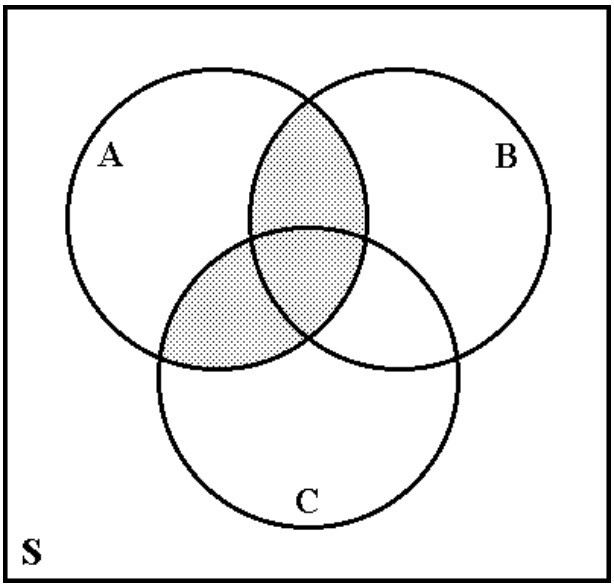
$P(A \cup B \cup C) =$ **0.75**.

b) Find $P((A \cap B) \cup C)$.



$P((A \cap B) \cup C) = \mathbf{0.67}$.

c) Find $P(A \cap (B \cup C))$.



$P(A \cap (B \cup C)) = \mathbf{0.37}$.