STAT 400 Fall 2017

Week 03 Discussion

1. ~ **1.5-11** ~ **1.6-11**

At the beginning of a certain study of a group of persons, 15% were classified as heavy smokers, 30% as light smokers, and 55% as nonsmokers. In the five-year study, it was determined that the death rates of the heavy smokers and light smokers were seven and three times that of the nonsmokers, respectively. A randomly selected participant died over the five-year period. What is the probability that the participant was a nonsmoker? A light smoker? A heavy smoker?

2. At a hospital's emergency room, patients are classified and 20% of them are critical, 30% are serious, and 50% are stable. Of the critical ones, 30% die; of the serious, 10% die; and of the stable, 2% die.

a) ~ **1.5-5** ~ **1.6-5** ~ 1.6-5

Given that a patient dies, what is the conditional probability that the patient was classified as critical? As serious? As stable?

b) Are events {a patient dies} and {a patient was classified as critical} independent? ***Justify your answer***.

c) Are events {a patient dies} and {a patient was classified as serious} independent? ***Justify your answer***.

3. Alex learns that his favorite soccer team, Urbana-Champaign United ($\cup\subset\cup$), has a 70% chance of signing one of the best players in the world, Ron Aldo. He immediately runs some computer simulations and discovers that if $\cup\subset\cup$ signs Ron Aldo, it would have a 0.90 probability of winning the American Central Illinois Division championship. Unfortunately, if $\cup\subset\cup$ does not sign Ron Aldo, then the probability of winning the championship is only 0.40. Alex becomes too excited and slips into a coma. He comes out of the coma a year later and finds out that $\cup\subset\cup$ has won the championship. What is the probability that $\cup\subset\cup$ was able to sign Ron Aldo?

4. Jack, Mike and Tom are roommates, and every Sunday night they split a large pizza for dinner. When there is only one slice left, the probability that Jack wants it is 0.40, the probability that Mike wants it is 0.35, and the probability that Tom wants it is 0.25. Suppose that whether or not each one of them will want the last slice is independent of the other two.

a) What is the probability that only one of the roommates will want the last slice?

b) What is the probability that at least one of the roommates will want the last slice?

c) What is the probability that at most one of the roommates will want the last slice?

5. Drug A is effective with probability 0.80. Drug B is effective with probability 0.70. There is a 40% chance of a negative drug interaction between drugs A and B. Suppose the effectiveness of the two drugs and the possibility of a negative drug interaction are all independent. Find the probability that …

a) … both drugs are effective and there is no negative drug interaction.

b) … at least one of the two drugs is effective and there is no negative drug interaction.

c) … at most one of the two drugs is effective and there is negative drug interaction.

6. A bank has two emergency sources of power for its computers. There is a 95% chance that source 1 will operate during a total power failure, and an 80% chance that source 2 will operate. Assume the power sources are independent. What is the probability that at least one of them will operate during a total power failure?

7. **1.4-2** **1.5-2** 1.5-2

If $P(A) = 0.3$, $P(B) = 0.6$.

a) Find $P(A \cup B)$ when A and B are independent.

b) Find $P(A \mid B)$ when A and B are mutually exclusive.

8. **1.4-5** **1.5-5** 1.5-5

If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$, are A and B independent events? Why or why not?

9. **1.4-8** **1.5-8** 1.5-8

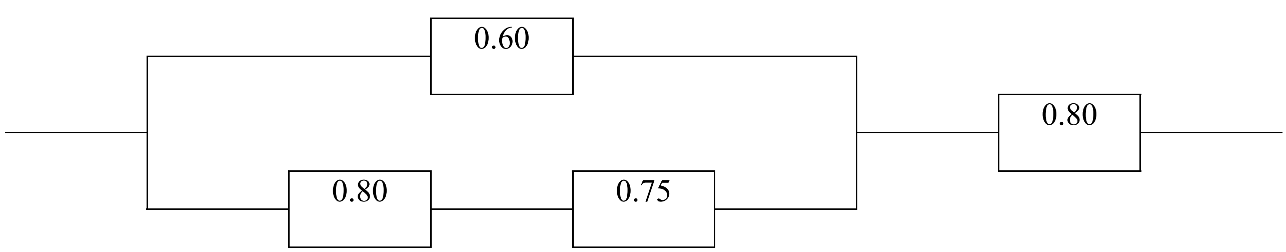
Die A has orange on one face and blue on five faces, Die B has orange on two faces and blue on four faces, Die C has orange on three faces and blue on three faces. All are fair dice. If the three dice are rolled, find the probability that exactly two of the three dice come up orange?

10. An electronic device has four independent components. Two of those four are new, and have a reliability of 0.80 each, one is old, with 0.75 reliability, and one is very old, and its reliability is 0.60.

a) Suppose that the device works if all four components are functional. What is the probability that the device will work when needed?

b) Suppose that the device works if at least one of the four components is functional. What is the probability that the device will work when needed?

c) Suppose that the four components are connected as shown on the diagram below. Find the reliability of the system.



11. An oral final exam continues until a student either answers two questions in a row correctly (and passes) or answers two questions in a row incorrectly (and fails). Suppose Alex has probability p to answer any question correctly, independently of any other questions. What is the probability that Alex would pass the exam?

12. Suppose Jane has a fair 4-sided die, and Dick has a fair 6-sided die. Each day, they roll their dice at the same time (independently) until someone rolls a "1" (as many times as necessary). (Then the person who did not roll a "1" does the dishes.) Find the probability that Jane rolls the first "1" before Dick does.

13. Prove (show) that

a) **1.2-10** **1.3-14** 1.3-14

$$\binom{n-1}{r} + \binom{n-1}{r-1} = \binom{n}{r}$$

(Pascal's equation).

b)

$$\binom{n-1}{r} - \binom{n-1}{r-1} = \frac{n-2r}{n}\binom{n}{r}.$$

14. We already know that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. Prove (show) that $\sum_{k=0}^{n} k\binom{n}{k} = n\,2^{n-1}$.

15. **1.2-4** **1.3-6** 1.3-6

The "eating club" is hosting a make-your-own sundae party at which the following are provided:

Ice Cream Flavors	Toppings
Chocolate	Caramel
Cookies 'n' cream	Hot fudge
Strawberry	Marshmallow
Vanilla	M&M's
	Nuts
	Strawberries

a) How many sundaes are possible using one flavor of ice cream and three different toppings?

b) How many sundaes are possible using one flavor of ice cream and from zero to six toppings?

c) How many different combinations of flavors of three scoops of ice cream are possible if it is permissible to make all three scoops the same flavor?

Answers:

1. ~ **1.5-11** ~ **1.6-11**

At the beginning of a certain study of a group of persons, 15% were classified as heavy smokers, 30% as light smokers, and 55% as nonsmokers. In the five-year study, it was determined that the death rates of the heavy smokers and light smokers were seven and three times that of the nonsmokers, respectively. A randomly selected participant died over the five-year period. What is the probability that the participant was a nonsmoker? A light smoker? A heavy smoker?

$P(\mathrm{H}) = 0.15, \qquad P(\mathrm{H}) = 0.30, \qquad P(\mathrm{N}) = 0.55,$

$P(\mathrm{D} \mid \mathrm{H}) = 7p, \qquad P(\mathrm{D} \mid \mathrm{L}) = 3p, \qquad P(\mathrm{D} \mid \mathrm{N}) = p.$

$$P(\mathrm{D}) = P(\mathrm{H}) \times P(\mathrm{D} \mid \mathrm{H}) + P(\mathrm{L}) \times P(\mathrm{D} \mid \mathrm{L}) + P(\mathrm{N}) \times P(\mathrm{D} \mid \mathrm{N})$$

$$= 0.15 \times 7p + 0.30 \times 3p + 0.55 \times p = 1.05p + 0.90p + 0.55p = 2.50p.$$

$$P(\mathrm{H} \mid \mathrm{D}) = \frac{1.05\,p}{2.50\,p} = \mathbf{0.42}.$$

$$P(\mathrm{L} \mid \mathrm{D}) = \frac{0.90\,p}{2.50\,p} = \mathbf{0.36}.$$

$$P(\mathrm{N} \mid \mathrm{D}) = \frac{0.55\,p}{2.50\,p} = \mathbf{0.22}.$$

2. At a hospital's emergency room, patients are classified and 20% of them are critical, 30% are serious, and 50% are stable. Of the critical ones, 30% die; of the serious, 10% die; and of the stable, 2% die.

a) ~ **1.5-5** ~ **1.6-5** ~ 1.6-5

Given that a patient dies, what is the conditional probability that the patient was classified as critical? As serious? As stable?

$$P(\text{patient dies}) = 0.20 \times 0.30 + 0.30 \times 0.10 + 0.50 \times 0.02$$
$$= 0.06 + 0.03 + 0.01 = 0.10.$$

$$P(\text{critical} \mid \text{patient dies}) = \frac{0.06}{0.10} = \mathbf{0.60}.$$

$$P(\text{serious} \mid \text{patient dies}) = \frac{0.03}{0.10} = \mathbf{0.30}.$$

$$P(\text{stable} \mid \text{patient dies}) = \frac{0.01}{0.10} = \mathbf{0.10}.$$

b) Are events {a patient dies} and {a patient was classified as critical} independent? ***Justify your answer***.

$$P(\text{patient dies} \cap \text{critical}) = 0.06 \neq 0.10 \times 0.20 = P(\text{patient dies}) \times P(\text{critical}).$$
$$P(\text{patient dies} \mid \text{critical}) = 0.30 \neq 0.10 = P(\text{patient dies}).$$
$$P(\text{critical} \mid \text{patient dies}) = 0.60 \neq 0.20 = P(\text{critical}).$$

{a patient dies} and {a patient was classified as critical} are **NOT independent**.

c) Are events {a patient dies} and {a patient was classified as serious} independent? ***Justify your answer***.

$$P(\text{patient dies} \cap \text{serious}) = 0.03 = 0.10 \times 0.30 = P(\text{patient dies}) \times P(\text{serious}).$$
$$P(\text{patient dies} \mid \text{serious}) = 0.10 = 0.10 = P(\text{patient dies}).$$
$$P(\text{serious} \mid \text{patient dies}) = 0.30 = 0.30 = P(\text{serious}).$$

{a patient dies} and {a patient was classified as critical} are **independent**.

3. Alex learns that his favorite soccer team, Urbana-Champaign United ($\cup\subset\cup$), has a 70% chance of signing one of the best players in the world, Ron Aldo. He immediately runs some computer simulations and discovers that if $\cup\subset\cup$ signs Ron Aldo, it would have a 0.90 probability of winning the American Central Illinois Division championship. Unfortunately, if $\cup\subset\cup$ does not sign Ron Aldo, then the probability of winning the championship is only 0.40. Alex becomes too excited and slips into a coma. He comes out of the coma a year later and finds out that $\cup\subset\cup$ has won the championship. What is the probability that $\cup\subset\cup$ was able to sign Ron Aldo?

$P(RA) = 0.70,\qquad P(W \mid RA) = 0.90,\qquad P(W \mid RA') = 0.40.$

Bayes' Theorem:

$$P(RA \mid W) = \frac{P(RA) \times P(W \mid RA)}{P(RA) \times P(W \mid RA) + P(RA') \times P(W \mid RA')}$$

$$= \frac{0.70 \times 0.90}{0.70 \times 0.90 + 0.30 \times 0.40} = \frac{0.63}{0.63 + 0.12} = \frac{0.63}{0.75} = \mathbf{0.84}.$$

OR

	W	W′	
RA	$0.90 \cdot 0.70$ 0.63	0.07	0.70
RA′	$0.40 \cdot 0.30$ 0.12	0.18	0.30
	0.75	0.25	1.00

$$P(RA \mid W) = \frac{0.63}{0.75} = \mathbf{0.84}.$$

4. Jack, Mike and Tom are roommates, and every Sunday night they split a large pizza for dinner. When there is only one slice left, the probability that Jack wants it is 0.40, the probability that Mike wants it is 0.35, and the probability that Tom wants it is 0.25. Suppose that whether or not each one of them will want the last slice is independent of the other two.

(Jack) = 0.40,	P(Jack$'$) = 0.60,
(Mike) = 0.35,	P(Mike$'$) = 0.65,
(Tom) = 0.25,	P(Tom$'$) = 0.75.

a) What is the probability that only one of the roommates will want the last slice?

only Jack	Jack	Mike$'$	Tom$'$	$0.40 \times 0.65 \times 0.75 = 0.1950$,
only Mike	Jack$'$	Mike	Tom$'$	$0.60 \times 0.35 \times 0.75 = 0.1575$,
only Tom	Jack$'$	Mike$'$	Tom	$0.60 \times 0.65 \times 0.25 = 0.0975$.

P(only one wants the last slice) = P(only Jack **or** only Mike **or** only Tom)

= P(only Jack) + P(only Mike) + P(only Tom)

= 0.1950 + 0.1575 + 0.0975 = **0.45**.

b) What is the probability that at least one of the roommates will want the last slice?

P(at least one wants the last slice) = 1 − P(no one wants the last slice)

= 1 − P(Jack$'$ $\cap$ Mike$'$ $\cap$ Tom$'$)

= $1 - 0.60 \times 0.65 \times 0.75$

= 1 − 0.2925 = **0.7075**.

c) What is the probability that at most one of the roommates will want the last slice?

P(at most one wants the last slice)

= P(only one wants the last slice) + P(no one wants the last slice)

= 0.45 + 0.2925 = **0.7425**.

5. Drug A is effective with probability 0.80. Drug B is effective with probability 0.70. There is a 40% chance of a negative drug interaction between drugs A and B. Suppose the effectiveness of the two drugs and the possibility of a negative drug interaction are all independent. Find the probability that …

a) … both drugs are effective and there is no negative drug interaction.

$$(\text{Drug A is effective})\text{ AND }(\text{Drug B is effective})\text{ AND }(\text{no negative drug interaction})$$

$$0.80 \times 0.70 \times 0.60 \;=\; \mathbf{0.336}.$$

b) … at least one of the two drugs is effective and there is no negative drug interaction.

$$[\,0.80 + 0.70 - 0.80 \times 0.70\,] \times 0.60 \;=\; \mathbf{0.564}.$$

OR

$$[\,1 - (\,1 - 0.80\,) \times (\,1 - 0.70\,)\,] \times 0.60 \;=\; \mathbf{0.564}.$$

OR

$$0.80 \times 0.70 \times 0.60 \;+\; 0.80 \times 0.30 \times 0.60 \;+\; 0.20 \times 0.70 \times 0.60$$
$$=\; 0.336 + 0.144 + 0.084 \;=\; \mathbf{0.564}.$$

c) … at most one of the two drugs is effective and there is negative drug interaction.

$$[\,1 - 0.80 \times 0.70\,] \times 0.40 \;=\; \mathbf{0.176}.$$

OR

$$0.20 \times 0.30 \times 0.40 \;+\; 0.80 \times 0.30 \times 0.40 \;+\; 0.20 \times 0.70 \times 0.40$$
$$=\; 0.024 + 0.096 + 0.056 \;=\; \mathbf{0.176}.$$

6. A bank has two emergency sources of power for its computers. There is a 95% chance that source 1 will operate during a total power failure, and an 80% chance that source 2 will operate. Assume the power sources are independent. What is the probability that at least one of them will operate during a total power failure?

$0.95 + 0.80 - 0.95 \times 0.80 \;=\; 0.95 + 0.80 - 0.76 \;=\; \mathbf{0.99}.$

OR $\quad 0.95 + 0.05 \times 0.80 \;=\; \mathbf{0.99}.$

OR $\quad 1 - 0.05 \times 0.20 \;=\; \mathbf{0.99}.$

7. **1.4-2** **1.5-2** 1.5-2

If $P(A) = 0.3,\; P(B) = 0.6.$

a) Find $P(A \cup B)$ when A and B are independent.

$P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.6 = 0.18;$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.18 = \mathbf{0.72}.$

b) Find $P(A \mid B)$ when A and B are mutually exclusive.

$P(A \cap B) = 0;$ $\qquad P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = \mathbf{0}.$

8. **1.4-5** **1.5-5** 1.5-5

If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$, are A and B independent events? Why or why not?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B). \qquad 0.90 = 0.80 + 0.50 - P(A \cap B).$$

$$\Rightarrow \qquad P(A \cap B) = 0.40.$$

$$P(A \cap B) = 0.40 \;\boldsymbol{=}\; 0.80 \times 0.50 = P(A) \times P(B).$$

$\Rightarrow$ A and B are **independent**.

9. **1.4-8** **1.5-8** 1.5-8

Die A has orange on one face and blue on five faces, Die B has orange on two faces and blue on four faces, Die C has orange on three faces and blue on three faces. All are fair dice. If the three dice are rolled, find the probability that exactly two of the three dice come up orange?

$$P(O\,O\,B) + P(O\,B\,O) + P(B\,O\,O) = \frac{1}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} + \frac{5}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \boldsymbol{\frac{2}{9}}.$$

10. An electronic device has four independent components. Two of those four are new, and have a reliability of 0.80 each, one is old, with 0.75 reliability, and one is very old, and its reliability is 0.60.

Let $A_i = \{ i^{\text{th}} \text{ component is functional} \}$.

Then $P(A_1) = P(A_2) = 0.80$, $P(A_3) = 0.75$, $P(A_4) = 0.60$.

a) Suppose that the device works if all four components are functional. What is the probability that the device will work when needed?

"all four" = "1st **and** 2nd **and** 3rd **and** 4th" = intersection.

$P(A_1 \cap A_2 \cap A_3 \cap A_4)$ since the components are independent

$= P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot P(A_4) = (0.80) \cdot (0.80) \cdot (0.75) \cdot (0.60) = \mathbf{0.288}$.

b) Suppose that the device works if at least one of the four components is functional. What is the probability that the device will work when needed?

"at least one" = "either 1st **or** 2nd **or** 3rd **or** 4th **or** 5th" = union.

P(at least one) = 1 – P(none).

"none" = "not 1st **and** not 2nd **and** not 3rd **and** not 4th **and** not 5th".

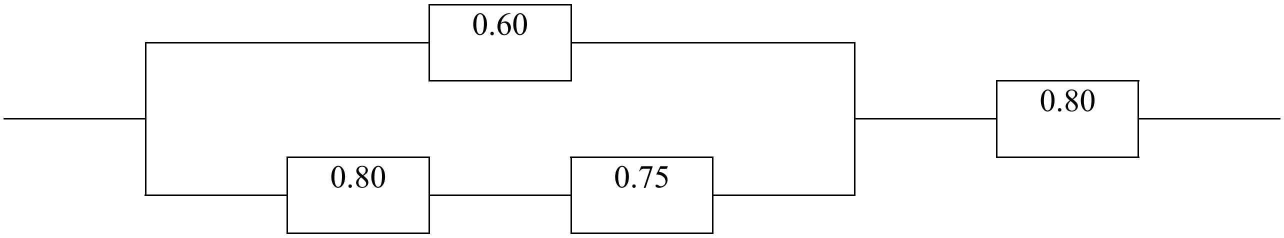
$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 1 - P(A_1' \cap A_2' \cap A_3' \cap A_4')$

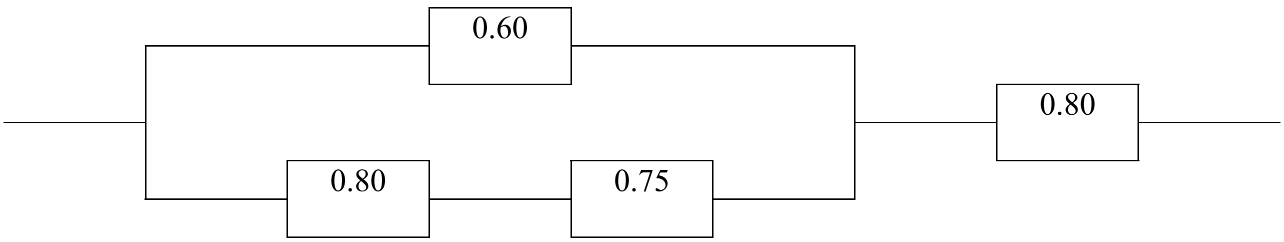
since the companies are independent

$= 1 - P(A_1') \cdot P(A_2') \cdot P(A_3') \cdot P(A_4')$

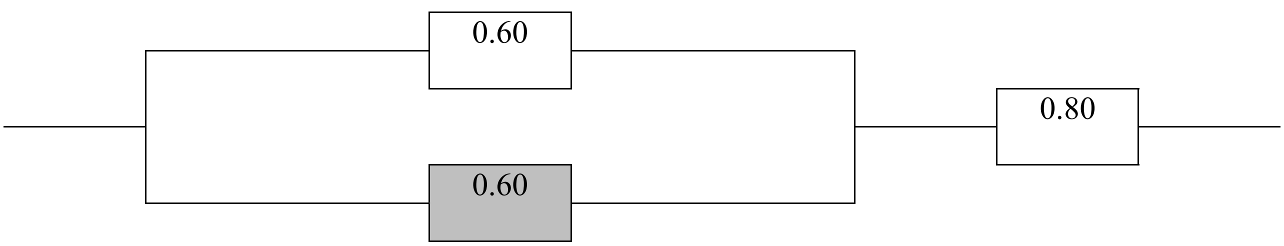
$= 1 - (0.20) \cdot (0.20) \cdot (0.25) \cdot (0.40) = 1 - 0.004 = \mathbf{0.996}$.

c) Suppose that the four components are connected as shown on the diagram below. Find the reliability of the system.

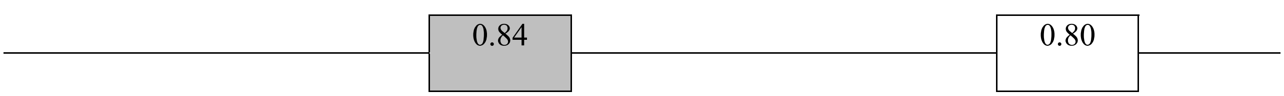




$0.80 \times 0.75 = 0.60$.



$1 - 0.40 \times 0.40 = 0.84$. OR $0.60 + 0.60 - 0.60 \times 0.60 = 0.84$.

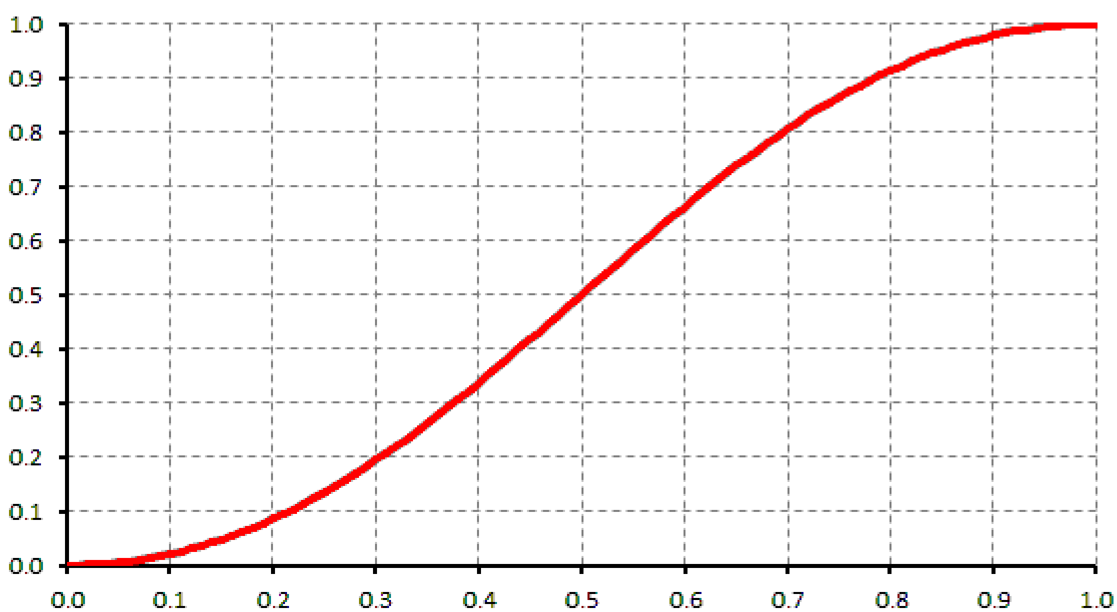


$0.84 \times 0.80 = \mathbf{0.672}$.

11. An oral final exam continues until a student either answers two questions in a row correctly (and passes) or answers two questions in a row incorrectly (and fails). Suppose Alex has probability p to answer any question correctly, independently of any other questions. What is the probability that Alex would pass the exam?

C C	W C C
C W C C	W C W C C
C W C W C C	W C W C W C C
• • •	• • •
$\Sigma\,(\mathrm{C\,W})^k\,\mathrm{C\,C}$	$\Sigma\,(\mathrm{W\,C})^k\,\mathrm{W\,C\,C}$
$\dfrac{p^2}{1-p\cdot(1-p)}$	$\dfrac{(1-p)\cdot p^2}{1-(1-p)\cdot p}$

$$\frac{p^2}{1-p\cdot(1-p)} + \frac{(1-p)\cdot p^2}{1-(1-p)\cdot p} = \frac{(2-p)\cdot p^2}{1-p\cdot(1-p)}.$$



12. Suppose Jane has a fair 4-sided die, and Dick has a fair 6-sided die. Each day, they roll their dice at the same time (independently) until someone rolls a "1" (as many times as necessary). (Then the person who did not roll a "1" does the dishes.) Find the probability that Jane rolls the first "1" before Dick does.

$$(J\,D') \quad \text{or} \quad (J'\,D')\,(J\,D') \quad \text{or} \quad (J'\,D')\,(J'\,D')\,(J\,D') \quad \text{or} \quad \ldots$$

$$\left(\frac{1}{4}\times\frac{5}{6}\right) + \left(\frac{3}{4}\times\frac{5}{6}\right)\times\left(\frac{1}{4}\times\frac{5}{6}\right) + \left(\frac{3}{4}\times\frac{5}{6}\right)\times\left(\frac{3}{4}\times\frac{5}{6}\right)\times\left(\frac{1}{4}\times\frac{5}{6}\right) + \ldots$$

$$= \sum_{k=0}^{\infty}\left(\frac{3}{4}\times\frac{5}{6}\right)^{k}\times\left(\frac{1}{4}\times\frac{5}{6}\right) = \frac{\left(\frac{1}{4}\times\frac{5}{6}\right)}{1-\left(\frac{3}{4}\times\frac{5}{6}\right)} = \mathbf{\frac{5}{9}}.$$

OR

1 turn	D	D′
J	$\frac{1}{4}\times\frac{1}{6} = \frac{1}{24}$ no one does dishes	$\frac{1}{4}\times\frac{5}{6} = \frac{5}{24}$ Dick does dishes
J′	$\frac{3}{4}\times\frac{1}{6} = \frac{3}{24}$ Jane does dishes	$\frac{3}{4}\times\frac{5}{6} = \frac{15}{24}$ game continues

$$\frac{1}{24}+\frac{5}{24}+\frac{3}{24} = \frac{9}{24}$$

game ends

$$\frac{5}{24}\Big/\frac{9}{24} = \mathbf{\frac{5}{9}}$$

Jane wins

13. Prove (show) that

a) **1.2-10** **1.3-14** 1.3-14

$$\binom{n-1}{r}+\binom{n-1}{r-1} = \binom{n}{r}$$

(Pascal's equation).

$$\binom{n-1}{r}+\binom{n-1}{r-1} = \frac{(n-1)!}{r!\cdot(n-r-1)!}+\frac{(n-1)!}{(r-1)!\cdot(n-r)!}$$

$$= \frac{(n-1)!}{(r-1)!\cdot(n-r-1)!}\cdot\left[\frac{1}{r}+\frac{1}{n-r}\right]$$

$$= \frac{(n-1)!}{(r-1)!\cdot(n-r-1)!}\cdot\frac{n}{r\cdot(n-r)} = \frac{n!}{r!\cdot(n-r)!} = \binom{n}{r}.$$

b) $$\binom{n-1}{r}-\binom{n-1}{r-1} = \frac{n-2r}{n}\binom{n}{r}.$$

$$\binom{n-1}{r}-\binom{n-1}{r-1} = \frac{(n-1)!}{r!\cdot(n-r-1)!}-\frac{(n-1)!}{(r-1)!\cdot(n-r)!}$$

$$= \frac{(n-1)!}{(r-1)!\cdot(n-r-1)!}\cdot\left[\frac{1}{r}-\frac{1}{n-r}\right]$$

$$= \frac{(n-1)!}{(r-1)!\cdot(n-r-1)!}\cdot\frac{n-2r}{r\cdot(n-r)}$$

$$= \frac{(n-1)!}{r!\cdot(n-r)!}\cdot(n-2r) = \frac{n!}{r!\cdot(n-r)!}\cdot\frac{n-2r}{n} = \frac{n-2r}{n}\binom{n}{r}.$$

14. We already know that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. Prove (show) that $\sum_{k=0}^{n} k \binom{n}{k} = n\,2^{n-1}$.

$$\sum_{k=0}^{n} k \binom{n}{k} = \sum_{k=1}^{n} k \binom{n}{k} = \sum_{k=1}^{n} k \frac{n!}{k! \cdot (n-k)!} = \sum_{k=1}^{n} \frac{n!}{(k-1)! \cdot (n-k)!}$$

$$= n \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)! \cdot (n-k)!} = n \sum_{m=0}^{n-1} \frac{(n-1)!}{m! \cdot (n-1-m)!}$$

$$= n \sum_{m=0}^{n-1} \binom{n-1}{m} = n\,2^{n-1}.$$

15. **1.2-4** **1.3-6** 1.3-6

The “eating club” is hosting a make-your-own sundae party at which the following are provided:

Ice Cream Flavors	**Toppings**
Chocolate	Caramel
Cookies ‘n’ cream	Hot fudge
Strawberry	Marshmallow
Vanilla	M&M’s
	Nuts
	Strawberries

a) How many sundaes are possible using one flavor of ice cream and three different toppings?

$$4 \times \binom{6}{3} = \mathbf{80}.$$

b) How many sundaes are possible using one flavor of ice cream and from zero to six toppings?

$$4 \times 2^6 = \mathbf{256}.$$

c) How many different combinations of flavors of three scoops of ice cream are possible if it is permissible to make all three scoops the same flavor?

The number of unordered selections of 3 objects that can be made out of 4 objects (repetitions are allowed) is $\binom{4-1+3}{3} = \binom{6}{3} = \mathbf{20}.$

1 1 1	1 2 3	2 2 2	2 4 4
1 1 2	1 2 4	2 2 3	3 3 3
1 1 3	1 3 3	2 2 4	3 3 4
1 1 4	1 3 4	2 3 3	3 4 4
1 2 2	1 4 4	2 3 4	4 4 4