Examples for 1.1

**Complement** of \( A \)

\[ A' \]

(not \( A, \bar{A}, A^c \))

contains all elements

that are **not** in \( A \)

**Intersection** of \( A \) and \( B \)

\[ A \cap B \]

(\( A \) and \( B, A B \))

contains all elements

that are in \( A \) **and** in \( B \)

**Union** of \( A \) and \( B \)

\[ A \cup B \]

(\( A \) or \( B \))

contains all elements

that are either in \( A \) **or** in \( B \)

or both

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**Axiom 1** Let \( A \) be any event defined over \( S \). Then \( P(A) \geq 0 \).

**Axiom 2** \( P(S) = 1 \).

**Axiom 3** If \( A_1, A_2, A_3, \ldots \) are events and \( A_i \cap A_j = \emptyset \) for each \( i \neq j \), then

\[ P(A_1 \cup A_2 \cup \ldots \cup A_k) = P(A_1) + P(A_2) + \ldots + P(A_k) \]

for each positive integer \( k \), and

\[ P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots \]

for an infinite, but countable, number of events.
Theorem 1. \[ P(A') = 1 - P(A). \]

Theorem 2. \[ P(\emptyset) = 0. \]

Theorem 3. If \( A \subset B \), then \( P(A) \leq P(B) \).

Theorem 4. For any event \( A \), \( P(A) \leq 1 \).

For any event \( A \), \( 0 \leq P(A) \leq 1 \)

\[ P(S) = 1, \] where \( S \) is the sample space.

Theorem 5.
If \( A \) and \( B \) are any two events, then
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B). \]
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B). \]

Theorem 6. \[ P(A \cup B \cup C) = P(A) + P(B) + P(C) \]
\[ - P(A \cap B) - P(A \cap C) - P(B \cap C) \]
\[ + P(A \cap B \cap C) \]

\[ P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D) \]
\[ - P(A \cap B) - P(A \cap C) - P(A \cap D) \]
\[ - P(B \cap C) - P(B \cap D) - P(C \cap D) \]
\[ + P(A \cap B \cap C) + P(A \cap B \cap D) \]
\[ + P(A \cap C \cap D) + P(B \cap C \cap D) \]
\[ - P(A \cap B \cap C \cap D) \]
1. Suppose a 6-sided die is rolled. The sample space, \( S \), is \( \{1, 2, 3, 4, 5, 6\} \). Consider the following events:
   \( A = \{ \text{the outcome is even} \} \),
   \( B = \{ \text{the outcome is greater than 3} \} \).

a) List outcomes in \( A \), \( B \), \( A' \), \( A \cap B \), \( A \cup B \).

b) Find the probabilities \( P(A) \), \( P(B) \), \( P(A') \), \( P(A \cap B) \), \( P(A \cup B) \) if the die is balanced (fair).

c) Suppose the die is loaded so that the probability of an outcome is proportional to the outcome, i.e.
   \( P(1) = p, \ P(2) = 2p, \ P(3) = 3p, \ P(4) = 4p, \ P(5) = 5p, \ P(6) = 6p. \)
   i) Find the value of \( p \) that would make this a valid probability model.

ii) Find the probabilities \( P(A) \), \( P(B) \), \( P(A') \), \( P(A \cap B) \), \( P(A \cup B) \).
2. Consider a “thick” coin with three possible outcomes of a toss (Heads, Tails, and Edge) for which Heads and Tails are equally likely, but Heads is five times as likely than Edge. What is the probability of Heads?

3. The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

a) What is the probability that a student selected at random does not own a bicycle?

b) What is the probability that a student selected at random owns either a car or a bicycle, or both?

c) What is the probability that a student selected at random has neither a car nor a bicycle?
4. During the first week of the semester, 80% of customers at a local convenience store bought either beer or potato chips (or both). 60% bought potato chips. 30% of the customers bought both beer and potato chips. What proportion of customers bought beer?

5. Suppose
\[ P(A) = 0.22, \]
\[ P(B) = 0.25, \]
\[ P(C) = 0.28, \]
\[ P(A \cap B) = 0.11, \]
\[ P(A \cap C) = 0.05, \]
\[ P(B \cap C) = 0.07, \]
\[ P(A \cap B \cap C) = 0.01. \]
Find the following:

a) \[ P(A \cup B) \]

b) \[ P(A' \cap B') \]

c) \[ P(A \cup B \cup C) \]

d) \[ P(A' \cap B' \cap C') \]

e) \[ P(A' \cap B' \cap C) \]

f) \[ P((A' \cap B') \cup C) \]

g) \[ P((A \cup B) \cap C) \]

h) \[ P((B \cap C') \cup A') \]
6. Let $a > 2$. Suppose $S = \{0, 1, 2, 3, \ldots\}$ and 

$$P(0) = c, \quad P(k) = \frac{1}{a^k}, \quad k = 1, 2, 3, \ldots.$$ 

a) Find the value of $c$ ($c$ will depend on $a$) that makes this a valid probability distribution.

b) Find the probability of an odd outcome.

7. Suppose $S = \{0, 1, 2, 3, \ldots\}$ and 

$$P(0) = p, \quad P(k) = \frac{1}{2^k \cdot k!}, \quad k = 1, 2, 3, \ldots.$$ 

Find the value of $p$ that would make this a valid probability model.