Answers for 1.3

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The **conditional probability of A, given B** (the probability of event A, computed on the assumption that event B has happened) is

$$\mathbf{P(A \mid B) = \frac{P(A \cap B)}{P(B)}} \qquad \text{(assuming } P(B) \neq 0 \text{).}$$

Similarly, the **conditional probability of B, given A** is

$$\mathbf{P(B \mid A) = \frac{P(A \cap B)}{P(A)}} \qquad \text{(assuming } P(A) \neq 0 \text{).}$$

3. (continued)

The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

	C	C′	
B	0.10	0.45	0.55
B′	0.20	0.25	0.45
	0.30	0.70	1.00

P(B) = 0.55, P(C) = 0.30, P(B ∩ C) = 0.10.

a) What is the probability that a student owns a bicycle, given that he/she owns a car?

$P(B \mid C) = {}^{0.10}/_{0.30} = \mathbf{{}^{1}/_{3}}$.

b) Suppose a student does not have a bicycle. What is the probability that he/she has a car?

$P(C \mid B') = {}^{0.20}/_{0.45} = \mathbf{{}^{4}/_{9}}$.

5. (continued)

Suppose

$P(A) = 0.22$,

$P(B) = 0.25$,

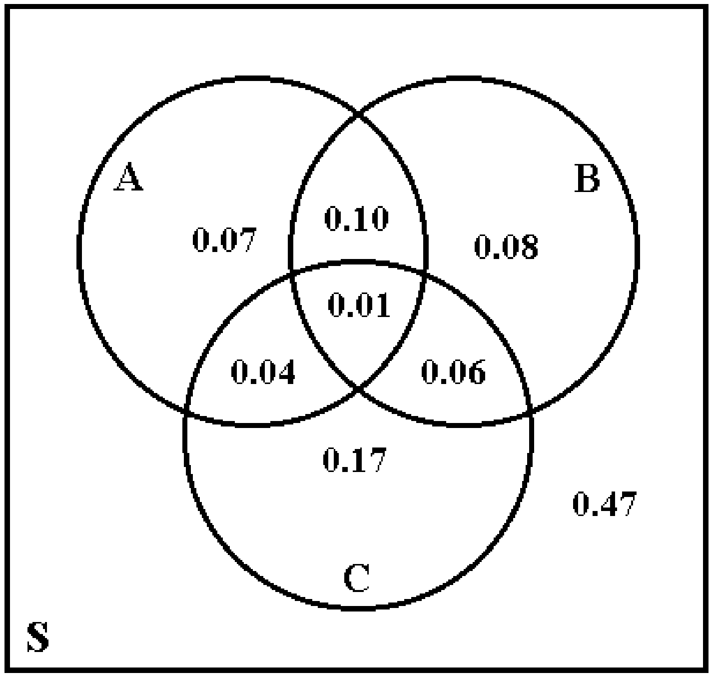
$P(C) = 0.28$,

$P(A \cap B) = 0.11$,

$P(A \cap C) = 0.05$,

$P(B \cap C) = 0.07$,

$P(A \cap B \cap C) = 0.01$.



Find the following:

a) $P(B \mid A)$

$P(B \mid A) = {}^{0.11}/_{0.22} = \mathbf{0.50}$.

b) $P(B \mid C)$

$P(B \mid C) = {}^{0.07}/_{0.28} = \mathbf{0.25}$.

c) $P(B \cap C \mid A)$

$P(B \cap C \mid A) = {}^{0.01}/_{0.22} = \mathbf{{}^{1}/_{22}}$.

d) $P(B \cup C \mid A)$

$P(B \cup C \mid A) = {}^{0.15}/_{0.22} = \mathbf{{}^{15}/_{22}}$.

e) $P(C \mid A \cup B)$

$P(C \mid A \cup B) = {}^{0.11}/_{0.36} = \mathbf{{}^{11}/_{36}}$.

f) $P(C \mid A \cap B)$

$P(C \mid A \cap B) = {}^{0.01}/_{0.11} = \mathbf{{}^{1}/_{11}}$.

g) $P(A \cap B \cap C \mid A \cup B \cup C)$

$P(A \cap B \cap C \mid A \cup B \cup C) = {}^{0.01}/_{0.53} = \mathbf{{}^{1}/_{53}}$.

Multiplication Law of Probability

If A and B are any two events, then

$$\mathbf{P(A \cap B) = P(A) \cdot P(B \mid A)}$$

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8. It is known that 30% of all the students at Anytown College live off campus. Suppose also that 48% of all the students are females. Of the female students, 25% live off campus.

P(Off) = 0.30, P(F) = 0.48, P(Off | F) = 0.25.

a) What is the probability that a randomly selected student is a female and lives off campus?

$P(F \cap Off) = P(F) \times P(Off | F) = 0.48 \times 0.25 =$ **0.12**.

	Off	On	
F	0.12	0.36	0.48
M	0.18	0.34	0.52
	0.30	0.70	1.00

b) What is the probability that a randomly selected student either is a female or lives off campus, or both?

$P(F \cup Off) = P(F) + P (Off) - P(F \cap Off) = 0.48 + 0.30 - 0.12 =$ **0.66**.

OR

$P(F \cup Off) = P(F \cap Off) + P(F' \cap Off) + P(F \cap Off')$

$= 0.12 + 0.18 + 0.36 =$ **0.66**.

OR

$P(F \cup Off) = 1 - P(F' \cap Off') = 1 - 0.34 =$ **0.66**.

c) What proportion of the off-campus students are females?

$P(F | Off) = {}^{0.12}/_{0.30} =$ **0.40**.

d) What proportion of the male students live off campus?

$P(Off | M) = {}^{0.18}/_{0.52} = {}^{9}/_{26} \approx$ **0.346154**.

9. Suppose that Joe's Discount Store has received a shipment of 25 television sets, 5 of which are defective. On the following day, 2 television sets are sold.

a) Find the probability that both of the television sets are defective.

P(both defective) = P(1st D $\cap$ 2nd D) = P(1st D) $\times$ P(2nd D | 1st D)

$$= \frac{5}{25} \times \frac{4}{24} = \mathbf{\frac{1}{30}}.$$

b) Find the probability that at least one of the two television sets sold is defective.

✓	D	D	$\frac{5}{25} \times \frac{4}{24} = \frac{1}{30}.$
✓	D	D′	$\frac{5}{25} \times \frac{20}{24} = \frac{5}{30}.$
✓	D′	D	$\frac{20}{25} \times \frac{5}{24} = \frac{5}{30}.$
×	D′	D′	

P(at least one D) = $\frac{1}{30} + \frac{5}{30} + \frac{5}{30} = \mathbf{\frac{11}{30}}.$

OR

P(at least one D) = $1 - P(D'D') = 1 - \frac{20}{25} \times \frac{19}{24} = 1 - \frac{19}{30} = \mathbf{\frac{11}{30}}.$

10. Cards are drawn one-by-one **without** replacement from a standard 52-card deck. What is the probability that …

a) … both the first and the second card drawn are ♥'s?

P(1st ♥ $\cap$ 2nd ♥) = P(1st ♥) $\times$ P(2nd ♥ | 1st ♥) = $\frac{13}{52} \times \frac{12}{51} = \mathbf{\frac{1}{17}}.$

b) … the first two cards drawn are a ♥ and a ♣ (or a ♣ and a ♥)?

$$P(\text{1st } \heartsuit \cap \text{2nd } \clubsuit) + P(\text{1st } \heartsuit \cap \text{2nd } \clubsuit) = {}^{13}/_{52} \times {}^{13}/_{51} + {}^{13}/_{52} \times {}^{13}/_{51}.$$

c) … there are at least two ♥'s among the first three cards drawn?

♥	♥	~~♥~~	$^{13}/_{52} \times {}^{12}/_{51} \times {}^{39}/_{50}$
	or		+
♥	~~♥~~	♥	$^{13}/_{52} \times {}^{39}/_{51} \times {}^{12}/_{50}$
	or		+
~~♥~~	♥	♥	$^{39}/_{52} \times {}^{13}/_{51} \times {}^{12}/_{50}$
	or		+
♥	♥	♥	$^{13}/_{52} \times {}^{12}/_{51} \times {}^{11}/_{50}$

$^{19968}/_{132600} \approx$ **0.150588**.