1. In Neverland, men constitute 60% of the labor force. The rates of unemployment are 6.0% and 4.5% among males and females, respectively. A person is selected at random from Neverland’s labor force.

\[ P(\text{M}) = 0.60, \quad P(\text{U} | \text{M}) = 0.06, \quad P(\text{U} | \text{F}) = 0.045. \]

a) What is the probability that the person selected is a male and is unemployed?

\[ P(\text{M} \cap \text{U}) = P(\text{M}) \times P(\text{U} | \text{M}) = 0.60 \times 0.06 = 0.036. \]

b) What is the probability that the person selected is a female and is unemployed?

\[ P(\text{F} \cap \text{U}) = P(\text{F}) \times P(\text{U} | \text{F}) = 0.40 \times 0.045 = 0.018. \]

<table>
<thead>
<tr>
<th></th>
<th>Unemployed</th>
<th>Employed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>0.036</td>
<td>0.564</td>
<td>0.60</td>
</tr>
<tr>
<td>Female</td>
<td>0.018</td>
<td>0.382</td>
<td>0.40</td>
</tr>
<tr>
<td>Total</td>
<td>0.054</td>
<td>0.946</td>
<td>1.00</td>
</tr>
</tbody>
</table>

c) What is the probability that the person selected is unemployed?

\[ P(\text{U}) = 0.036 + 0.018 = 0.054. \]

OR

Law of Total Probability:

\[ P(\text{U}) = P(\text{M}) \times P(\text{U} | \text{M}) + P(\text{F}) \times P(\text{U} | \text{F}) = 0.60 \times 0.06 + 0.40 \times 0.045 = 0.054. \]

d) Suppose the person selected is unemployed. What is the probability that a male was selected?

\[ P(\text{M} | \text{U}) = \frac{0.036}{0.054} = \frac{2}{3}. \]

OR

Bayes’ Theorem:

\[ P(\text{M} | \text{U}) = \frac{P(\text{M}) \times P(\text{U} | \text{M})}{P(\text{M}) \times P(\text{U} | \text{M}) + P(\text{F}) \times P(\text{U} | \text{F})} = \frac{0.60 \times 0.06}{0.60 \times 0.06 + 0.40 \times 0.045} = \frac{2}{3}. \]
2. In a presidential race in Neverland, the incumbent Democrat \((D)\) is running against a field of four Republicans \((R_1, R_2, R_3, R_4)\) seeking the nomination. Political pundits estimate that the probabilities of \(R_1, R_2, R_3,\) and \(R_4\) winning the nomination are 0.40, 0.30, 0.20, and 0.10, respectively. Furthermore, results from a variety of polls are suggesting that \(D\) would have a 55% chance of defeating \(R_1\) in the general election, a 70% chance of defeating \(R_2\), a 60% chance of defeating \(R_3\), and an 80% chance of defeating \(R_4\). Assuming all these estimates to be accurate, what are the chances that \(D\) will be a two-term president?

\[
\begin{align*}
P(R_1) &= 0.40, \quad P(R_2) = 0.30, \quad P(R_3) = 0.20, \quad P(R_4) = 0.10, \\
P(W | R_1) &= 0.55, \quad P(W | R_2) = 0.70, \quad P(W | R_3) = 0.60, \quad P(W | R_4) = 0.80.
\end{align*}
\]

Law of Total Probability:

\[
P(W) = P(W \cap R_1) + P(W \cap R_2) + P(W \cap R_3) + P(W \cap R_4)
\]

\[
= P(R_1)P(W | R_1) + P(R_2)P(W | R_2) + P(R_3)P(W | R_3) + P(R_4)P(W | R_4)
\]

\[
= 0.40 \cdot 0.55 + 0.30 \cdot 0.70 + 0.20 \cdot 0.60 + 0.10 \cdot 0.80 = 0.63.
\]
3. In Anytown, 10% of the people leave their keys in the ignition of their cars. Anytown’s police records indicate that 4.2% of the cars with keys left in the ignition are stolen. On the other hand, only 0.2% of the cars without keys left in the ignition are stolen. **Suppose a car in Anytown is stolen.** What is the probability that the keys were left in the ignition?

\[
P( \text{Keys}) = 0.10, \quad P( \text{Keys}') = 1 - 0.10 = 0.90.
\]

\[
P( \text{Stolen} \mid \text{Keys}) = 0.042. \quad P( \text{Stolen} \mid \text{Keys}') = 0.002.
\]

Need \( P( \text{Keys} \mid \text{Stolen}) = ? \)

**Bayes’ Theorem:**

\[
P(\text{Keys} \mid \text{Stolen}) = \frac{P(\text{Keys}) \times P(\text{Stolen} \mid \text{Keys})}{P(\text{Keys}) \times P(\text{Stolen} \mid \text{Keys}) + P(\text{Keys}') \times P(\text{Stolen} \mid \text{Keys}')} = \frac{0.10 \times 0.042}{0.10 \times 0.042 + 0.90 \times 0.002} = 0.70.
\]

**OR**

<table>
<thead>
<tr>
<th></th>
<th>Stolen</th>
<th>Stolen’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keys</td>
<td>0.042</td>
<td>0.0958</td>
</tr>
<tr>
<td>Keys’</td>
<td>0.002</td>
<td>0.8982</td>
</tr>
<tr>
<td></td>
<td>0.0060</td>
<td>0.9940</td>
</tr>
</tbody>
</table>

\[
P( \text{Keys} \mid \text{Stolen}) = \frac{P(\text{Keys} \cap \text{Stolen})}{P(\text{Stolen})} = \frac{0.0042}{0.0060} = 0.70.
\]

**OR**
\[
P(\text{Stolen}) = 0.0042 + 0.0018 = 0.0060.\]

\[
P(\text{Keys} \mid \text{Stolen}) = \frac{P(\text{Keys} \cap \text{Stolen})}{P(\text{Stolen})} = \frac{0.0042}{0.0060} = 0.70.\]
A warehouse receives widgets from three different manufacturers, A (50%), B (30%), and C (20%). Suppose that 2% of the widgets coming from A are defective, as are 4% of the widgets coming from B, and 7% coming from C.

a) Find the probability that a widget selected at random at this warehouse is defective.

Law of Total Probability:

\[
P(D) = P(A) \times P(D|A) + P(B) \times P(D|B) + P(C) \times P(D|C)
\]

\[
= 0.50 \times 0.02 + 0.30 \times 0.04 + 0.20 \times 0.07 \\
= 0.010 + 0.012 + 0.014 = 0.036.
\]

b) Suppose a widget that was selected at random is found to be defective. What is the probability that it came from manufacturer A? Manufacturer B? Manufacturer C?

\[
P(A|D) = \frac{P(A) \times P(D|A)}{P(D)} = \frac{0.010}{0.036} = \frac{5}{18}.
\]

\[
P(B|D) = \frac{P(B) \times P(D|B)}{P(D)} = \frac{0.012}{0.036} = \frac{6}{18}.
\]

\[
P(C|D) = \frac{P(C) \times P(D|C)}{P(D)} = \frac{0.014}{0.036} = \frac{7}{18}.
\]
3½. Seventy percent of the light aircraft that disappear while in flight in Neverland are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft that has just disappeared has an emergency locator. What is the probability that it will not be discovered?

\[
P(\text{Discovered}) = 0.70, \quad P(\text{Discovered}') = 1 - 0.70 = 0.30.
\]

\[
P(\text{Locator} \mid \text{Discovered}) = 0.60. \quad P(\text{Locator} ' \mid \text{Discovered}') = 0.90.
\]

Need \( P(\text{Discovered}' \mid \text{Locator}) = ? \)

<table>
<thead>
<tr>
<th></th>
<th>Locator</th>
<th>Locator'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discovered</td>
<td>0.60 \cdot 0.70</td>
<td>0.42</td>
</tr>
<tr>
<td>Discovered'</td>
<td>0.03 \cdot 0.90</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>0.55</td>
</tr>
</tbody>
</table>

\[
P(\text{Discovered}' \mid \text{Locator}) = \frac{P(\text{Discovered}' \cap \text{Locator})}{P(\text{Locator})} = \frac{0.03}{0.45} = \frac{1}{15} \approx 0.06667.
\]
4. In a certain population, the proportion of individuals who have a particular disease is 0.025. A test for the disease is positive in 94% of the people who have the disease and in 4% of the people who do not.

\[ P(D) = 0.025, \quad P(+|D) = 0.94, \quad P(+|D') = 0.04. \]

**a)** Find the probability of receiving a positive reaction from this test.

Need \( P(+) = ? \)

<table>
<thead>
<tr>
<th></th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>( 0.025 \cdot 0.94 )</td>
<td>( 0.0015 )</td>
</tr>
<tr>
<td>( D' )</td>
<td>( 0.975 \cdot 0.04 )</td>
<td>( 0.9360 )</td>
</tr>
<tr>
<td></td>
<td>( \textbf{0.0625} )</td>
<td>( 0.9375 )</td>
</tr>
</tbody>
</table>

OR

Law of Total Probability:

\[ P(+) = P(D) \times P(+) \mid D + P(D') \times P(+) \mid D' = 0.025 \times 0.94 + 0.975 \times 0.04 = \textbf{0.0625}. \]

**b)** If a person received a positive reaction from this test, what is the probability that he/she has the disease?

\[ P(D \mid +) = \frac{0.0235}{0.0625} = \textbf{0.376}. \]

OR

Bayes’ Theorem:

\[ P(D \mid +) = \frac{0.025 \times 0.94}{0.025 \times 0.94 + 0.975 \times 0.04} = \textbf{0.376}. \]

**c)** If a person received a negative reaction from this test, what is the probability that he/she doesn’t have the disease?

\[ P(D' \mid -) = \frac{0.9360}{0.9375} = \textbf{0.9984}. \]

OR

Bayes’ Theorem:

\[ P(D' \mid -) = \frac{0.975 \times 0.96}{0.025 \times 0.06 + 0.975 \times 0.96} = \textbf{0.9984}. \]