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Examples for 2.1, 2.2, 2.3

A **random variable** associates a numerical value with each outcome of a random experiment.

A random variable is said to be **discrete** if it has either a finite number of values or infinitely many values that can be arranged in a sequence.

If a random variable represents some measurement on a continuous scale and therefore capable of assuming all values in an interval, it is called a **continuous** random variable.

The **probability distribution** of a discrete random variable is a list of all its distinct numerical values along with their associated probabilities:

x	$f(x)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$
.	.
.	.
.	.
x_n	$f(x_n)$
	1.00

! 1) for each x,

$$0 \le f(x) \le 1.$$

! 2) $\sum_{\text{all } x} f(x) = 1.$

Often a formula can be used in place of a detailed list.

1. A balanced (fair) coin is tossed twice. Let X denote the number of H's. Construct the probability distribution of X.

x	$f(x)$

2. Suppose a random variable X has the following probability distribution:

x	$f(x)$
10	0.20
11	0.40
12	0.30
13	0.10

a) Find the expected value of X, E(X).

x	$f(x)$	$x \cdot f(x)$
10	0.2	
11	0.4	
12	0.3	
13	0.1	

$$E(X) = \mu_X = \sum_{\text{all } x} x \cdot f(x)$$

b) Find the variance of X, Var(X).

x	$f(x)$	$(x - \mu_X)$	$(x - \mu_X)^2 \cdot f(x)$
10	0.2		
11	0.4		
12	0.3		
13	0.1		

$$\mathrm{Var}(X) = \sigma_X^2 = \sum_{\text{all } x} (x - \mu_X)^2 \cdot f(x)$$

$$= E[(X - \mu_X)^2]$$

x	$f(x)$	$x^2 \cdot f(x)$
10	0.2	
11	0.4	
12	0.3	
13	0.1	

$$\text{Var(X)} = \sigma_X^2 = \sum_{\text{all } x} x^2 \cdot f(x) - [\text{E(X)}]^2$$

$$= \text{E}(\text{X}^2) - [\text{E(X)}]^2$$

c) Find the standard deviation of X, SD(X).

$$\text{SD(X)} = \sigma_X = \sqrt{\sigma_X^2}$$

d) Find the cumulative distribution function of X, $\text{F}(x) = \text{P}(\text{X} \le x)$.

x	$f(x)$	$\text{F}(x)$
10	0.2	
11	0.4	
12	0.3	
13	0.1	

$$\mathrm{E}\big(g(\mathrm{X})\big) = \sum_{\text{all } x} g(x) \cdot f(x)$$

$$\mathbf{E}\,(\boldsymbol{a} \cdot \mathrm{X} + \boldsymbol{b}) \;=\; \boldsymbol{a} \cdot \mathbf{E}\,(\mathrm{X}) + \boldsymbol{b}.$$

$$\mathbf{Var}\,(\boldsymbol{a} \cdot \mathrm{X} + \boldsymbol{b}) \;=\; \boldsymbol{a}^{2} \cdot \mathbf{Var}\,(\mathrm{X}).$$

$$\mathbf{SD}\,(\boldsymbol{a} \cdot \mathrm{X} + \boldsymbol{b}) \;=\; |\boldsymbol{a}| \cdot \mathbf{SD}\,(\mathrm{X}).$$

3. Suppose E(X) = 7, SD(X) = 3.

a) Y = 2 X + 3. Find E(Y) and SD(Y).

b) W = 5 – 2 X. Find E(W) and SD(W).

4. Suppose a discrete random variable X has the following probability distribution:

$$\mathrm{P}(\,\mathrm{X} = 0\,) = 2 - \sqrt{e}\,, \qquad \mathrm{P}(\,\mathrm{X} = k\,) = \frac{1}{2^{k} \cdot k!}, \quad k = 1, 2, 3, \ldots$$

a) Find $\mathrm{E}\,(\,\mathrm{X}\,)$.

b) Find $\mathrm{Var}\,(\,\mathrm{X}\,)$.