Examples for 2.6

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Poisson Distribution:

X = the number of occurrences of a particular event in an interval of time or space.

$$P(X = x) = \frac{\lambda^{x} \cdot e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, 3, \ldots .$$

$$E(X) = \lambda, \qquad \mathrm{Var}(X) = \lambda.$$

Table III (pp. 580 – 582) gives $P(X \le x)$

EXCEL: $= \mathrm{POISSON}(x, \lambda, 0)$ gives $P(X = x)$

$= \mathrm{POISSON}(x, \lambda, 1)$ gives $P(X \le x)$

1. Traffic accidents at a particular intersection follow Poisson distribution with an average rate of 1.4 per week.

a) What is the probability that the next week is accident-free?

b) What is the probability that there will be exactly 3 accidents next week?

c) What is the probability that there will be at most 2 accidents next week?

d) What is the probability that there will be at least 2 accidents during the next two weeks?

e) What is the probability that there will be exactly 5 accidents during the next four weeks?

f) What is the probability that there will be exactly 2 accidents tomorrow?

g) What is the probability that the next accident will not occur for three days?

h) What is the probability that there will be exactly three accident-free weeks during the next eight weeks?

i) What is the probability that there will be exactly five accident-free days during the next week?

When n is large ($n \geq 20$) and p is small ($p \leq 0.05$) and $n \cdot p \leq 5$, Binomial probabilities can be approximated by Poisson probabilities. For this, set $\lambda = n \cdot p$.

2. Suppose the defective rate at a particular factory is 1%. Suppose 50 parts were selected from the daily output of parts. Let X denote the number of defective parts in the sample.

a) Find the probability that the sample contains exactly 2 defective parts.

b) Use Poisson approximation to find the probability that the sample contains exactly 2 defective parts.

c) Find the probability that the sample contains at most 1 defective part.

d) Use Poisson approximation to find the probability that the sample contains at most 1defective part.