Poisson Distribution:

X = the number of occurrences of a particular event in an interval of time or space.

\[ P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \ldots. \]

E(X) = \lambda, \quad \text{Var}(X) = \lambda.

Table III (pp. 580 – 582) gives \( P(X \leq x) \)

1. Traffic accidents at a particular intersection follow Poisson distribution with an average rate of 1.4 per week.

a) What is the probability that the next week is accident-free?

1 week \( \Rightarrow \lambda = 1.4. \)

\[ P(X = 0) = \frac{1.4^0 e^{-1.4}}{0!} \approx 0.2466. \]

b) What is the probability that there will be exactly 3 accidents next week?

1 week \( \Rightarrow \lambda = 1.4. \)

\[ P(X = 3) = \frac{1.4^3 e^{-1.4}}{3!} \approx 0.1128. \]

c) What is the probability that there will be at most 2 accidents next week?

1 week \( \Rightarrow \lambda = 1.4. \)

\[ P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) \]
\[ = \frac{1.4^0 e^{-1.4}}{0!} + \frac{1.4^1 e^{-1.4}}{1!} + \frac{1.4^2 e^{-1.4}}{2!} \]
\[ \approx 0.2466 + 0.3452 + 0.2417 = 0.8335. \]
d) What is the probability that there will be at least 2 accidents during the next two weeks?

2 weeks \( \Rightarrow \lambda = 2.8. \)

\[ P(X \geq 2) = 1 - \left[ P(X = 0) + P(X = 1) \right] = 1 - \left[ \frac{2.8^0 \cdot e^{-2.8}}{0!} + \frac{2.8^1 \cdot e^{-2.8}}{1!} \right] \]

\[ \approx 1 - [0.0608 + 0.1703] = 0.7689. \]

e) What is the probability that there will be exactly 5 accidents during the next four weeks?

4 weeks \( \Rightarrow \lambda = 5.6. \)

\[ P(X = 5) = \frac{5.6^5 \cdot e^{-5.6}}{5!} \approx 0.1697. \]

f) What is the probability that there will be exactly 2 accidents tomorrow?

1 day \( \Rightarrow \lambda = 0.2. \)

\[ P(X = 2) = \frac{0.2^2 \cdot e^{-0.2}}{2!} \approx 0.0164. \]

g) What is the probability that the next accident will not occur for three days?

3 days \( \Rightarrow \lambda = 0.6. \)

\[ P(X = 0) = \frac{0.6^0 \cdot e^{-0.6}}{0!} \approx 0.5488. \]

h) What is the probability that there will be exactly three accident-free weeks during the next eight weeks?

\begin{align*}
\text{“Success”} & \quad = \text{an accident-free week} \\
1 \text{ week} \ & \Rightarrow \lambda = 1.4. \quad \quad \quad \quad p = P(\text{“Success”}) = P(X = 0) = \frac{1.4^0 \cdot e^{-1.4}}{0!} \approx 0.2466. \\
\end{align*}

\[ P(\text{exactly 3 accident-free weeks in 8 weeks}) = \binom{8}{3} \cdot 0.2466^3 \cdot 0.7534^5 \approx 0.20384. \]

( Binomial distribution )
i) What is the probability that there will be exactly five accident-free days during the next week?

“Success” = an accident-free day

\[ 1 \text{ day} \Rightarrow \lambda = 0.2. \quad p = P(\text{“Success”}) = P(X = 0) = \frac{0.2^0 \cdot e^{-0.2}}{0!} \approx 0.81873. \]

\[ P(\text{exactly 5 accident-free days in 7 days}) = 7C_5 \cdot 0.81873^5 \cdot 0.18127^2 \approx 0.25385. \text{ (Binomial distribution)} \]

When \( n \) is large (\( n \geq 20 \)) and \( p \) is small (\( p \leq 0.05 \)) and \( n \cdot p \leq 5 \), Binomial probabilities can be approximated by Poisson probabilities. For this, set \( \lambda = n \cdot p \).

2. Suppose the defective rate at a particular factory is 1%. Suppose 50 parts were selected from the daily output of parts. Let \( X \) denote the number of defective parts in the sample.

a) Find the probability that the sample contains exactly 2 defective parts.

\[ P(X = 2) = \binom{50}{2} \cdot (0.01)^2 \cdot (0.99)^{48} \approx 0.075618. \]

b) Use Poisson approximation to find the probability that the sample contains exactly 2 defective parts.

\[ \lambda = n \cdot p = 0.5. \]

\[ P(X = 2) = \frac{0.5^2 \cdot e^{-0.5}}{2!} \approx 0.075816. \]
c) Find the probability that the sample contains at most 1 defective part.

\[
P(X \leq 1) = P(X = 0) + P(X = 1)
\]

\[
= \binom{50}{0} \cdot (0.01)^0 \cdot (0.99)^{50} + \binom{50}{1} \cdot (0.01)^1 \cdot (0.99)^{49} \approx 0.910565.
\]

d) Use Poisson approximation to find the probability that the sample contains at most 1 defective part.

\[
P(X \leq 1) = P(X = 0) + P(X = 1)
\]

\[
= \frac{0.5^0 \cdot e^{-0.5}}{0!} + \frac{0.5^1 \cdot e^{-0.5}}{1!} \approx 0.909796.
\]