Examples for 4.1, 4.4 (Part 1)

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Multivariate Distributions

Let X and Y be two discrete random variables. The **joint probability mass function** $p(x, y)$ is defined for each pair of numbers (x, y) by

$$p(x, y) = \mathrm{P}(\mathrm{X} = x \text{ and } \mathrm{Y} = y).$$

Let A be any set consisting of pairs of (x, y) values. Then

$$\mathrm{P}\big((\mathrm{X}, \mathrm{Y}) \in A\big) = \sum_{(x,y) \in A} \sum p(x, y).$$

Let X and Y be two continuous random variables. Then $f(x, y)$ is the **joint probability density function** for X and Y if for any two-dimensional set A

$$\mathrm{P}\big((\mathrm{X}, \mathrm{Y}) \in A\big) = \iint_A f(x, y)\, dx\, dy.$$

1. Consider the following joint probability distribution $p(x, y)$ of two random variables X and Y:

x \ y	0	1	2	
1	0.15	0.10	0	
2	0.25	0.30	0.20	

a) Find $\mathrm{P}(\mathrm{X} + \mathrm{Y} = 2)$.

b) Find $\mathrm{P}(\mathrm{X} > \mathrm{Y})$.

The **marginal probability mass functions** of X and of Y are given by

$$p_{\text{X}}(x) = \sum_{\text{all } y} p(x, y), \qquad p_{\text{Y}}(y) = \sum_{\text{all } x} p(x, y).$$

The **marginal probability density functions** of X and of Y are given by

$$f_{\text{X}}(x) = \int_{-\infty}^{\infty} f(x, y)\,dy, \qquad f_{\text{Y}}(y) = \int_{-\infty}^{\infty} f(x, y)\,dx.$$

c) Find the (marginal) probability distributions $p_{\text{X}}(x)$ of X and $p_{\text{Y}}(y)$ of Y.

x	$p_{\text{X}}(x)$

y	$p_{\text{Y}}(y)$

If $p(x, y)$ is the joint probability mass function of (X, Y) OR $f(x, y)$ is the joint probability density function of (X, Y), then

discrete

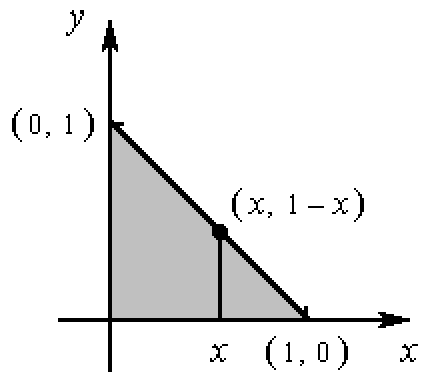
$$\text{E}(g(\text{X}, \text{Y})) = \sum_{\text{all } x} \sum_{\text{all } y} g(x, y) \cdot p(x, y)$$

continuous

$$\text{E}(g(\text{X}, \text{Y})) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y)\,dx\,dy$$

d) Find $\text{E}(\text{X}), \text{E}(\text{Y}), \text{E}(\text{X} + \text{Y}), \text{E}(\text{X} \cdot \text{Y})$.

2. Alexis Nuts, Inc. markets cans of deluxe mixed nuts containing almonds, cashews, and peanuts. Suppose the net weight of each can is exactly 1 lb, but the weight contribution of each type of nut is random. Because the three weights sum to 1, a joint probability model for any two gives all necessary information about the weight of the third type. Let X = the weight of almonds in a selected can and Y = the weight of cashews.



Then the region of positive density is D = $\{ (x, y) : 0 \le x \le 1, 0 \le y \le 1, x + y \le 1 \}$.

Let the joint probability density function for (X, Y) be

$$f(x,y) = \begin{cases} 60\,x^2\,y & 0 \le x \le 1,\ 0 \le y \le 1,\ x + y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Verify that $f(x,y)$ is a legitimate probability density function.

b) Find the probability that the two types of nuts together make up less than 50% of the can. That is, find the probability $P(X + Y < 0.50)$. (Find the probability that peanuts make up over 50% of the can.)

c) Find the probability that there are more almonds than cashews in a can. That is, find the probability $P(X > Y)$.

d) Find the probability that there are at least twice as many cashews as there are almonds. That is, find the probability $P(2X \le Y)$.

e) Find the marginal probability density function for X.

f) Find the marginal probability density function for Y.

g) Find $E(X)$, $E(Y)$, $E(X+Y)$, $E(X \cdot Y)$.

h) If 1 lb of almonds costs the company \$1.00, 1 lb of cashews costs \$1.50, and 1 lb of peanuts costs \$0.60, what is the expected total cost of the content of a can?