1. Suppose the lifetime of a particular brand of light bulbs is normally distributed with standard deviation of $\sigma = 75$ hours and unknown mean.

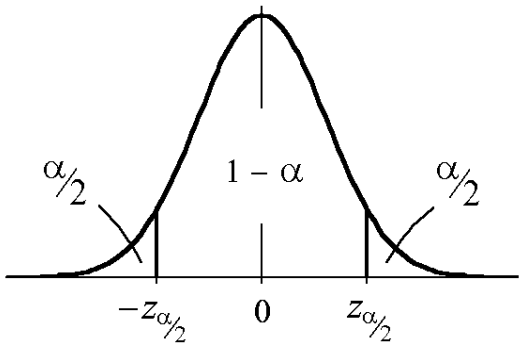
a) What is the probability that in a random sample of $n = 49$ bulbs, the average lifetime $\overline{X}$ is within 21 hours of the overall average lifetime?

b) Suppose the sample average lifetime of $n = 49$ bulbs is $\bar{x} = 843$ hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

A **confidence interval** is a *range of numbers* believed to include an unknown population parameter. Associated with the interval is a measure of the *confidence* we have that the interval does indeed contain the parameter of interest.

A $(1 - \alpha)$ 100% confidence interval for the population mean μ when σ is known and sampling is done from a normal population, or with a large sample, is

$$\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \, , \; \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$$



$$\bar{X} \quad \pm \quad z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \qquad\qquad \bar{X} \pm \varepsilon$$

estimate (point estimate) | margin of error ε | $\varepsilon = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

1. (continued)

Suppose the sample average lifetime of $n = 49$ bulbs is $\bar{x} = 843$ hours.

b) Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

c) Construct a 90% confidence interval for the overall average lifetime for light bulbs.

d) Construct a 92% confidence interval for the overall average lifetime for light bulbs.

Minimum required sample size in estimating the population mean μ to within ε with $(1-\alpha)100\%$ confidence is

$$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2 .$$

Always round n up.

2. How many test runs of an automobile are required for determining its average miles-per-gallon rating on the highway to within 0.5 miles per gallon with 95% confidence, if a guess is that the variance of the population of miles per gallon is about 6.25?

1. (continued)

e) What is the minimum sample size required if we wish to estimate the overall average lifetime for light bulbs to within 10 hours with 90% confidence?