1. Suppose the lifetime of a particular brand of light bulbs is normally distributed with standard deviation of $\sigma = 75$ hours and unknown mean.

a) What is the probability that in a random sample of 49 bulbs, the average lifetime $\bar{X}$ is within 21 hours of the overall average lifetime?

$\sigma = 75, \quad n = 49.$

$$P(\mu - 21 < \bar{X} < \mu + 21) = P\left(\frac{\mu - 21 - \mu}{75/\sqrt{49}} < Z < \frac{\mu + 21 - \mu}{75/\sqrt{49}}\right)$$

$$= P(-1.96 < Z < 1.96) = 0.95.$$ 

b) Suppose the sample average lifetime of the 49 bulbs is $\bar{x} = 843$ hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

$$(\bar{X} - 21, \bar{X} + 21) \quad (822, 864)$$

A confidence interval is a range of numbers believed to include an unknown population parameter. Associated with the interval is a measure of the confidence we have that the interval does indeed contain the parameter of interest.

A $(1 - \alpha)$ 100% confidence interval for the population mean $\mu$

when $\sigma$ is known

and sampling is done from a normal population, or with a large sample, is

$$\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$
1. (continued)

Suppose the sample average lifetime of the 49 bulbs is \( \bar{x} = 843 \) hours.

b) Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

\[
\sigma = 75 \text{ is known. } n = 49 \text{ – large. The confidence interval: } \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.
\]

95% confidence level, \( \alpha = 0.05 \), \( \alpha/2 = 0.025 \), \( z_{\alpha/2} = 1.96 \).

\[
843 \pm 1.96 \cdot \frac{75}{\sqrt{49}} = 843 \pm 21 = (822, 864)
\]

c) Construct a 90% confidence interval for the overall average lifetime for light bulbs.

90% confidence level, \( \alpha = 0.10 \), \( \alpha/2 = 0.05 \), \( z_{\alpha/2} = 1.645 \).

\[
843 \pm 1.645 \cdot \frac{75}{\sqrt{49}} = 843 \pm 17.625 = (825.375, 860.625)
\]

d) Construct a 92% confidence interval for the overall average lifetime for light bulbs.

92% confidence level, \( \alpha = 0.08 \), \( \alpha/2 = 0.04 \), \( z_{\alpha/2} = 1.75 \).

\[
843 \pm 1.75 \cdot \frac{75}{\sqrt{49}} = 843 \pm 18.75 = (824.25, 861.75)
\]
Minimum required sample size in estimating the population mean $\mu$ to within $\varepsilon$ with $(1 - \alpha) 100\%$ confidence is

$$n = \left[ \frac{Z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2.$$

Always round $n$ up.

2. How many test runs of an automobile are required for determining its average miles-per-gallon rating on the highway to within 0.5 miles per gallon with 95% confidence, if a guess is that the variance of the population of miles per gallon is about 6.25?

$$\varepsilon = 0.5, \quad \sigma^2 = 6.25, \quad \sigma = 2.5,$$

95% confidence level, $\alpha = 0.05$, $Z_{\alpha/2} = 1.960$.

$$n = \left[ \frac{Z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2 = \left[ \frac{1.96 \cdot 2.5}{0.5} \right]^2 = 96.04.$$

Round up. $n = 97$.

1. (continued)

e) What is the minimum sample size required if we wish to estimate the overall average lifetime for light bulbs to within 10 hours with 90% confidence?

$$\varepsilon = 10, \quad \sigma = 75,$$

90% confidence level, $\alpha = 0.10$, $Z_{\alpha/2} = 1.645$.

$$n = \left[ \frac{Z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2 = \left[ \frac{1.645 \cdot 75}{10} \right]^2 = 152.21390625.$$

Round up. $n = 153$. 