STAT 400 UIUC

Examples for 5.5, 7.1

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Let $X_1, X_2, \dots, X_n$ be i.i.d. $\mathbf{N}(\mu, \sigma^2)$.

Let

$$\overline{X} = \frac{\sum X_i}{n} = \frac{X_1 + X_2 + \dots + X_n}{n} \qquad \text{(sample mean)}$$

$$S^2 = \frac{\sum (X_i - \overline{X})^2}{n-1} \qquad \text{(sample variance)}$$

Then

$\overline{X}$ and S^2 are independent;

$\overline{X}$ has $\mathbf{N}\left(\mu, \frac{\sigma^2}{n}\right)$ distribution;

$\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$ has $\mathbf{N}(0,1)$ distribution;

$\dfrac{\sum (X_i - \mu)^2}{\sigma^2}$ has $\chi^2(n)$ distribution;

$\dfrac{(n-1) \cdot S^2}{\sigma^2} = \dfrac{\sum (X_i - \overline{X})^2}{\sigma^2}$ has $\chi^2(n-1)$ distribution;

$\dfrac{\overline{X} - \mu}{S / \sqrt{n}}$ has $t(n-1)$ distribution.

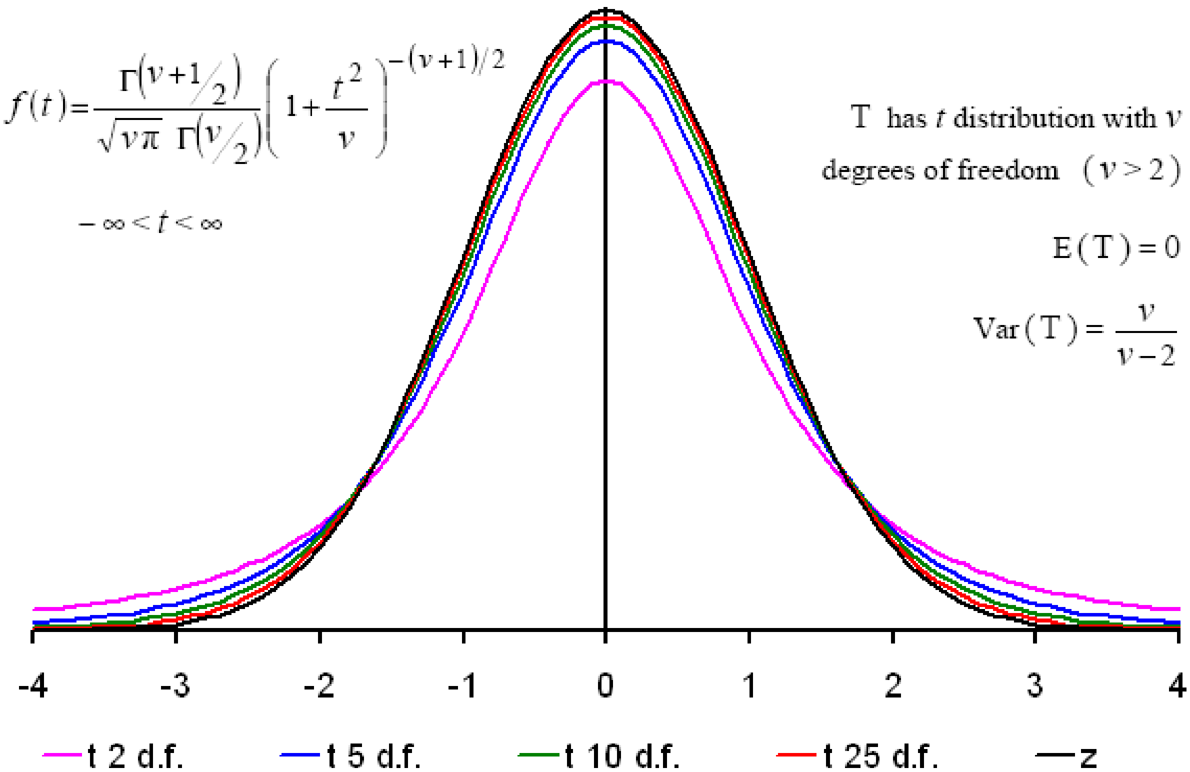
A $(1 - \alpha)$ 100% confidence interval for the population mean μ

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \qquad \qquad \overline{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$n - 1$ degrees of freedom

William Gosset
(1876-1937)

The t Distribution



EXCEL:

$=\text{TINV}(\alpha, v)$ gives $t_{\alpha/2}$ for t distribution with v degrees of freedom

$=\text{TDIST}(t, v, 1)$ gives the upper tail probability for t distribution with v degrees of freedom, $P(T > t)$.

$=\text{TDIST}(t, v, 2)$ gives $2 \times P(T > t)$.

1. A manufacturer of TV sets wants to find the average selling price of a particular model. A random sample of 25 different stores gives the mean selling price as $342 with a sample standard deviation of $14. Assume the prices are normally distributed. Construct a 95% confidence interval for the mean selling price of the TV model.

2. The following random sample was obtained from $N(\mu, \sigma^2)$ distribution:

16	12	18	13	21	15	8	17

a) Compute the sample mean and the sample standard deviation.

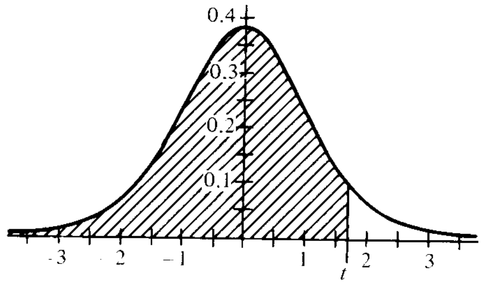
2. (continued)

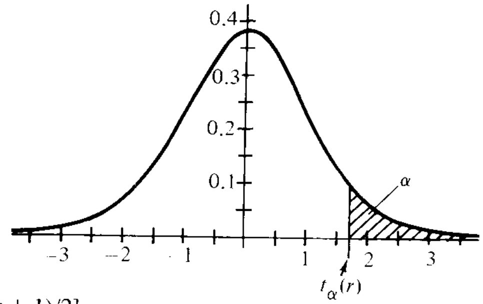
b) Construct a 95% confidence interval for μ.

c) Construct a 90% confidence upper bound for μ.

d) Construct a 99% confidence lower bound for μ.

Table VI
The t Distribution





$$P(T \le t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1+w^2/r)^{(r+1)/2}}\,dw$$

$$[P(T \le -t) = 1 - P(T \le t)]$$

	$P(T \le t)$						
	0.60	0.75	0.90	0.95	0.975	0.99	0.995
r	$t_{0.40}(r)$	$t_{0.25}(r)$	$t_{0.10}(r)$	$t_{0.05}(r)$	$t_{0.025}(r)$	$t_{0.01}(r)$	$t_{0.005}(r)$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576