1. A machine makes \( \frac{1}{2} \)-inch ball bearings. In a random sample of 41 bearings, the sample standard deviation of the diameters of the bearings was 0.02 inch. Assume that the diameters of the bearings are approximately normally distributed. Construct a 90% confidence interval for the standard deviation of the diameters of the bearings.

\( s = 0.02, \quad n = 41 \).

The confidence interval:

\[
\left( \sqrt{\frac{(n-1) \cdot s^2}{\chi^2_{\alpha/2}}}, \sqrt{\frac{(n-1) \cdot s^2}{\chi^2_{1-\alpha/2}}} \right)
\]

90% confidence level \( \alpha = 0.10 \)

\[
\frac{\alpha}{2} = 0.05.
\]

\( n - 1 = 40 \) degrees of freedom.

\[
\chi^2_{\alpha/2} = \chi^2_{0.05} = 55.76, \quad \chi^2_{1-\alpha/2} = \chi^2_{0.95} = 26.51.
\]

\[
\left( \sqrt{\frac{(41-1) \cdot 0.02^2}{55.76}}, \sqrt{\frac{(41-1) \cdot 0.02^2}{26.51}} \right)
\]

(0.01694, 0.02457)

2. The following random sample was obtained from \( N(\mu, \sigma^2) \) distribution:

\[
16 \quad 12 \quad 18 \quad 13 \quad 21 \quad 15 \quad 8 \quad 17
\]

Recall: \( \bar{x} = 15, \quad s^2 = 16, \quad s = 4 \).

a) Construct a 95% confidence interval for the overall standard deviation.

Confidence Interval for \( \sigma^2 \):

\[
\left( \sqrt{\frac{(n-1) \cdot s^2}{\chi^2_{\alpha/2}}}, \sqrt{\frac{(n-1) \cdot s^2}{\chi^2_{1-\alpha/2}}} \right)
\]
\[ \alpha = 0.05. \quad \alpha/2 = 0.025. \quad 1 - \alpha/2 = 0.975. \]

number of degrees of freedom = \( n - 1 = 8 - 1 = 7. \)

\[ \chi^2_{\alpha/2} = 16.01. \quad \chi^2_{1 - \alpha/2} = 1.690. \]

\[ \left( \frac{(8 - 1) \cdot 16}{16.01}, \frac{(8 - 1) \cdot 16}{1.690} \right) \quad \left( 6.9956 ; 66.2722 \right) \]

Confidence Interval for \( \sigma \):

\[ \left( \sqrt{6.9956} , \sqrt{66.2722} \right) = (2.645 ; 8.141) \]

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b) Construct a 95% confidence lower bound for the overall standard deviation.

\[ \left( \frac{(n - 1) \cdot s^2}{\chi^2_{\alpha}}, \infty \right) \quad 7 \text{ degrees of freedom} \quad \chi^2_{0.05} = 14.07. \]

\[ \left( \frac{(8 - 1) \cdot 16}{14.07}, \infty \right) \quad \left( 2.82 ; \infty \right) \]

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c) Construct a 95% confidence upper bound for the overall standard deviation.

95% conf. upper bound for \( \sigma \):

\[ 0, \sqrt{\frac{(n - 1) \cdot s^2}{\chi^2_{1 - \alpha}}} \quad = \quad 0, \sqrt{\frac{(8 - 1) \cdot 16}{2.167}} \quad = \quad (0, 7.19) \]