Hypotheses Testing for the population mean $\mu$

Null Alternative
$H_0 : \mu \geq \mu_0$ vs. $H_1 : \mu < \mu_0$ Left - tailed.
$H_0 : \mu \leq \mu_0$ vs. $H_1 : \mu > \mu_0$ Right - tailed.
$H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$ Two - tailed.

Test Statistic: $Z = \frac{X - \mu_0}{\sigma / \sqrt{n}}$ OR $T = \frac{X - \mu_0}{s / \sqrt{n}}$ OR $\bar{X}$

Rejection Region:

$H_0 : \mu \geq \mu_0$ $H_0 : \mu \leq \mu_0$ $H_0 : \mu = \mu_0$
$H_1 : \mu < \mu_0$ $H_1 : \mu > \mu_0$ $H_1 : \mu \neq \mu_0$
Left - tailed. Right - tailed. Two - tailed.

Reject $H_0$ if $Z < -z_\alpha$
Reject $H_0$ if $Z > z_\alpha$
Reject $H_0$ if $\bar{x} < \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}}$
Reject $H_0$ if $\bar{x} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$
$\bar{x} < \mu_0 - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $\bar{x} > \mu_0 + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
1. The overall standard deviation of the diameters of the ball bearings is $\sigma = 0.005$ mm. The overall mean diameter of the ball bearings must be 4.300 mm. A sample of 81 ball bearings had a sample mean diameter of 4.299 mm. Is there a reason to believe that the actual overall mean diameter of the ball bearings is not 4.300 mm?

a) Perform the appropriate test using a 10% level of significance.

Claim:

$H_0 : \mu = 4.300$ mm vs. $H_1 : \mu \neq 4.300$ mm

Test Statistic:

Rejection Region: 

P-value: 

Decision: 

Confidence Interval: 

b) State your decision (Accept $H_0$ or Reject $H_0$) for the significance level $\alpha = 0.05$. 

2. A trucking firm believes that its mean weekly loss due to damaged shipments is at most $1800. Half a year (26 weeks) of operation shows a sample mean weekly loss of $1921.54 with a sample standard deviation of $249.39.

a) Perform the appropriate test. Use the significance level $\alpha = 0.10$.

Claim:

$H_0 : \mu \leq 1800$ vs. $H_1 : \mu > 1800$

Test Statistic:

Rejection Region: $t > t_{0.10}$

P-value: $P(t > t_{0.10})$

Decision:

b) State your decision (Accept $H_0$ or Reject $H_0$) for the significance level $\alpha = 0.05$.

The t Distribution

<table>
<thead>
<tr>
<th>$t$</th>
<th>0.40</th>
<th>0.25</th>
<th>0.20</th>
<th>0.15</th>
<th>0.10</th>
<th>0.05</th>
<th>0.025</th>
<th>0.02</th>
<th>0.01</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.256</td>
<td>0.684</td>
<td>0.856</td>
<td>1.058</td>
<td>1.316</td>
<td>1.708</td>
<td>2.060</td>
<td>2.167</td>
<td>2.485</td>
<td>2.787</td>
</tr>
</tbody>
</table>
3. *Metaltech Industries* manufactures carbide drill tips used in drilling oil wells. The life of a carbide drill tip is measured by how many feet can be drilled before the tip wears out. *Metaltech* claims that under typical drilling conditions, the life of a carbide tip follows a normal distribution with mean of at least 32 feet. Suppose some customers disagree with *Metaltech*’s claims and argue that *Metaltech* is overstating the mean (i.e. the mean is actually less than 32). *Metaltech* agrees to examine a random sample of 25 carbide tips to test its claim against the customers’ claim. If the *Metaltech*’s claim is rejected, *Metaltech* has agreed to give customers a price rebate on past purchases. Suppose *Metaltech* decided to use a 5% level of significance and the observed sample mean is 30.5 feet with the sample variance 16 feet². Perform the appropriate test.

Claim:

\[ H_0 : \quad \text{vs.} \quad H_1 : \]

Test Statistic:

Rejection Region: | P-value:

Decision:

The t Distribution

<table>
<thead>
<tr>
<th></th>
<th>( t_{0.40} )</th>
<th>( t_{0.25} )</th>
<th>( t_{0.20} )</th>
<th>( t_{0.15} )</th>
<th>( t_{0.10} )</th>
<th>( t_{0.05} )</th>
<th>( t_{0.025} )</th>
<th>( t_{0.02} )</th>
<th>( t_{0.01} )</th>
<th>( t_{0.005} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.256</td>
<td>0.685</td>
<td>0.857</td>
<td>1.059</td>
<td>1.318</td>
<td>1.711</td>
<td>2.064</td>
<td>2.172</td>
<td>2.492</td>
<td>2.797</td>
</tr>
</tbody>
</table>