STAT 400
UIUC

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Examples for 8.1 (Part 1)

Hypotheses Testing for the population mean μ

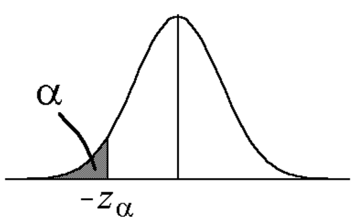
Null		Alternative	
$H_0: \mu \geq \mu_0$	vs.	$H_1: \mu < \mu_0$	Left - tailed.
$H_0: \mu \leq \mu_0$	vs.	$H_1: \mu > \mu_0$	Right - tailed.
$H_0: \mu = \mu_0$	vs.	$H_1: \mu \neq \mu_0$	Two - tailed.

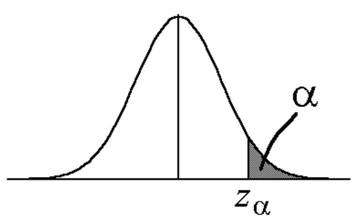
Test Statistic:

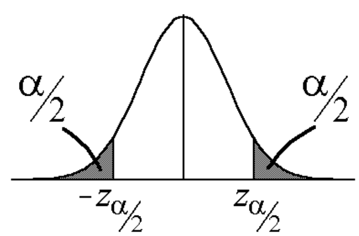
$$Z = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \qquad \text{OR} \qquad T = \frac{\overline{X} - \mu_0}{s / \sqrt{n}} \qquad \text{OR} \qquad \overline{X}$$

Rejection Region:

$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu = \mu_0$
$H_1: \mu < \mu_0$	$H_1: \mu > \mu_0$	$H_1: \mu \neq \mu_0$
Left - tailed.	Right - tailed.	Two - tailed.







Reject H_0 if

$$Z < -z_{\alpha}$$

Reject H_0 if

$$Z > z_{\alpha}$$

Reject H_0 if

$$Z < -z_{\alpha/2}$$

or

$$Z > z_{\alpha/2}$$

Reject H_0 if

$$\overline{x} < \mu_0 - z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

Reject H_0 if

$$\overline{x} > \mu_0 + z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

Reject H_0 if

$$\overline{x} < \mu_0 - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

or

$$\overline{x} > \mu_0 + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

1. The overall standard deviation of the diameters of the ball bearings is $\sigma = 0.005$ mm. The overall mean diameter of the ball bearings must be 4.300 mm. A sample of 81 ball bearings had a sample mean diameter of 4.299 mm. Is there a reason to believe that the actual overall mean diameter of the ball bearings is not 4.300 mm?

a) Perform the appropriate test using a 10% level of significance.

Claim:

H_0 : vs. H_1 :

Test Statistic:

Rejection Region:

Decision:

P-value:

Decision:

Confidence Interval:

Decision:

b) State your decision (Accept H_0 or Reject H_0) for the significance level $\alpha = 0.05$.

2. A trucking firm believes that its mean weekly loss due to damaged shipments is at most \$1800. Half a year (26 weeks) of operation shows a sample mean weekly loss of \$1921.54 with a sample standard deviation of \$249.39.

a) Perform the appropriate test. Use the significance level $\alpha = 0.10$.

Claim:

H_0 : vs. H_1 :

Test Statistic:

Rejection Region:

Decision:

P-value:

Decision:

b) State your decision (Accept H_0 or Reject H_0) for the significance level $\alpha = 0.05$.

The t Distribution

r	$t_{0.40}$	$t_{0.25}$	$t_{0.20}$	$t_{0.15}$	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.02}$	$t_{0.01}$	$t_{0.005}$
25	0.256	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787

3. *Metaltech Industries* manufactures carbide drill tips used in drilling oil wells. The life of a carbide drill tip is measured by how many feet can be drilled before the tip wears out. *Metaltech* claims that under typical drilling conditions, the life of a carbide tip follows a normal distribution with mean of at least 32 feet. Suppose some customers disagree with *Metaltech*'s claims and argue that *Metaltech* is overstating the mean (i.e. the mean is actually less than 32). *Metaltech* agrees to examine a random sample of 25 carbide tips to test its claim against the customers' claim. If the *Metaltech*'s claim is rejected, *Metaltech* has agreed to give customers a price rebate on past purchases. Suppose *Metaltech* decided to use a 5% level of significance and the observed sample mean is 30.5 feet with the sample variance 16 feet^2. Perform the appropriate test.

Claim:

H_0 : vs. H_1 :

Test Statistic:

Rejection Region:

Decision:

P-value:

Decision:

The t Distribution

r	$t_{0.40}$	$t_{0.25}$	$t_{0.20}$	$t_{0.15}$	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.02}$	$t_{0.01}$	$t_{0.005}$
24	0.256	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797