Examples for 9.2

Stepanov
Dalpiaz

The χ^2 Test of Homogeneity (one margin fixed)

Independent random samples from r populations.

Each sample classified in c response categories.

H_0: In each response category, the probabilities are equal for all r populations.

The χ^2 Test of Independence (neither margin fixed)

A random sample of size n is simultaneously classified with respect to two characteristics, one has r categories and the other c categories.

H_0: The two classifications are independent; that is, each cell probability is the product of the row and column marginal probabilities.

Test Statistic:

$$Q = \sum_{cells} \frac{(O - E)^2}{E} \qquad \begin{cases} O = \text{observed cell frequency} \\ E = \dfrac{\text{row total} \times \text{column total}}{\text{grand total}} \end{cases}$$

Rejection Region:

$$\text{Reject } H_0 \text{ if } Q \geq \chi^2_{\alpha},$$

$$\text{d.f.} = (\text{No. of rows} - 1) \times (\text{No. of columns} - 1) = (r - 1) \times (c - 1)$$

1. We wish to test whether the proportions of individuals with each of the four blood types are the same in two neighboring towns, Town X and Town Y. A random sample of 300individuals from Town X and 200 individuals from Town Y produced the following observed frequencies:

	Blood Type				
	O	A	B	AB	
Town X	120	85	60	35	300
Town Y	100	45	30	25	200
	220	130	90	60	500

Use $\alpha = 0.05$ to test $\mathrm{H}_0 : p_{\mathrm{XO}} = p_{\mathrm{YO}},\ p_{\mathrm{XA}} = p_{\mathrm{YA}},\ p_{\mathrm{XB}} = p_{\mathrm{YB}},\ p_{\mathrm{XAB}} = p_{\mathrm{YAB}}.$

2. In a random sample of 500 voters, each individual was asked whether he or she thought inflation of unemployment was a more serious problem. The individuals were also classified by party affiliation. The results were as follows:

Party	Unemployment	Inflation
Democrat	150	70
Republican	100	80
Other	60	40

Use a 5% level of significance and test whether political party affiliation and perceived problem are independent.

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09

3. In a comparative study of two new drugs, A and B, 120 patients were treated with drug A and 150 patients with drug B, and the following results were obtained.

	Drug A	Drug B
Cured	78	111
Not cured	42	39
Total	120	150

We wish to test whether drug A and drug B have the same cure rate.

$$\mathrm{H}_0: \; p_{\mathrm{AC}} = p_{\mathrm{BC}}, \quad p_{\mathrm{AN}} = p_{\mathrm{BN}}.$$

Recall: $\hat{p}_1 = \dfrac{\mathrm{Y}_1}{n_1} = \dfrac{78}{120} = 0.65.$ $\hat{p}_2 = \dfrac{\mathrm{Y}_2}{n_2} = \dfrac{111}{150} = 0.74.$

$$\hat{p} = \frac{\mathrm{Y}_1 + \mathrm{Y}_2}{n_1 + n_2} = \frac{78 + 111}{120 + 150} = \frac{189}{270} = 0.70.$$

Test Statistic: $$\mathrm{Z} = \frac{0.65 - 0.74}{\sqrt{0.70 \cdot 0.30 \cdot \left(\frac{1}{120} + \frac{1}{150} \right)}} \approx \mathbf{-1.60357}.$$