STAT 400
Exam I

Basic Probability

Fall 2017
Practice

1. A statistics instructor teaches two statistics classes back-to-back, lecture AL1 at 11:00 am and lecture CL1 at 12 noon. Suppose the probability that the instructor gives a good AL1 lecture is 0.25. If AL1 lecture is good, then CL1 lecture is also good with probability 0.50. However, if AL1 lecture is bad, then there is an 90% chance that CL1 lecture will also be bad.

a) What proportion of the CL1 lectures are good?

b) Suppose CL1 lecture was good. What is the probability that AL1 lecture was also good?

c) If at the conclusion of the CL1 lecture, the instructor is heard to mutter "what a rotten lecture," what is the probability that the AL1 lecture was also bad?

d) What is the probability that at least one lecture is good (i.e., either AL1 lecture is good or CL1 lecture is good, or both)?

2. **1.3-4 (c)** **1.4-4 (c)**

Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing a heart on the first draw and an ace on the second draw.

Hint: Note that a heart can be drawn by getting the ace of hearts or one of the other 12 hearts.

3. Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B.

a) **1.3-16** **1.4-18**

Compute the probability of then drawing a red chip from bowl B.

b) If a red chip was drawn from bowl B, what is the probability that a red chip had been drawn from bowl A?

4. Consider two events, A and B, such that $P(A) = 0.70$, $P(B) = 0.40$ and $P(A \cup B) = 0.82$. Are A and B independent?

5. At a hospital's emergency room, patients are classified and 20% of them are critical, 30% are serious, and 50% are stable. Of the critical ones, 30% die; of the serious, 10% die; and of the stable, 2% die.

a) ~ **1.5-5** ~ **1.6-5**

Given that a patient dies, what is the conditional probability that the patient was classified as critical? As serious? As stable?

b) Are events {a patient dies} and {a patient was classified as critical} independent? ***Justify your answer***.

c) Are events {a patient dies} and {a patient was classified as serious} independent? ***Justify your answer***.

6. Does a monkey have a better chance of rearranging

A C C L L S U U to spell C A L C U L U S

or

A A B E G L R to spell A L G E B R A ?

7. Three cards are drawn at random without replacement from a standard 52-card deck. What is the probability that either all three cards are face cards (king, queen, jack) or all three cards are red cards (♦, ♥) (or both)?

8. A laboratory has 15 rats, 7 white, 6 gray and 2 brown. Suppose 5 rats will be selected at random and assigned to an experimental drug.

a) Find the probability that 2 white, 2 gray and 1 brown rats were selected.

b) Find the probability that 3 white and 2 gray rats were selected.

c) Find the probability that all five rats selected have the same color.

d) Find the probability that all five rats selected have different color.

9. A bank has two emergency sources of power for its computers. There is a 95% chance that source 1 will operate during a total power failure, and an 80% chance that source 2 will operate. Assume the power sources are independent. What is the probability that at least one of them will operate during a total power failure?

10. If a fair 6-sided die is rolled 6 times, what is the probability that each possible outcome (1, 2, 3, 4, 5, and 6) will occur exactly once?

11. **1.3-7** **1.4-9**

An urn contains four colored balls: two orange and two blue. Two balls are selected at random without replacement, and you are told that at least one of them is orange. What is the probability that the other ball is also orange?

Hint 1: O O O B B O B B

Hint 2: $P(\text{both orange} \mid \text{at least one orange})$.

12. A new flavor of toothpaste has been developed. It was tested by a group of 15 people. Nine of the group said they liked the new flavor, and the remaining 6 indicated they did not. Five of the 15 are selected at random to participate in an in-depth interview.

a) What is the probability that of those selected for the in-depth interview 4 liked the new flavor and 1 did not?

b) What is the probability that of those selected for the in-depth interview at least 4 liked the new flavor?

13. A company that manufactures video cameras produces a basic model and a deluxe model. Suppose that 25% of the cameras sold are the basic model. Of those buying the basic model, 20% purchase an extended warranty, whereas 40% of the deluxe model purchasers do so.

a) What proportion of customers purchase an extended warranty?

b) What proportion of customers either purchase an extended warranty or purchase a deluxe model, or both?

c) If you learn that a randomly selected purchaser has an extended warranty, what is the probability that he/she has a deluxe model?

d) Suppose a randomly selected purchaser does not have an extended warranty. What is the probability that he/she has a basic model?

1. A statistics instructor teaches two statistics classes back-to-back, lecture AL1 at 11:00 am and lecture CL1 at 12 noon. Suppose the probability that the instructor gives a good AL1 lecture is 0.25. If AL1 lecture is good, then CL1 lecture is also good with probability 0.50. However, if AL1 lecture is bad, then there is an 90% chance that CL1 lecture will also be bad.

A = {good AL1 lecture}, C = {good CL1 lecture}.

$P(A) = 0.25$, $P(C \mid A) = 0.50$, $P(C' \mid A') = 0.90$.

a) What proportion of the CL1 lectures are good?

$$P(C) = P(A \cap C) + P(A' \cap C) = P(A) \cdot P(C \mid A) + P(A') \cdot P(C \mid A')$$
$$= 0.25 \cdot 0.50 + 0.75 \cdot 0.10 = 0.125 + 0.075 = \mathbf{0.20}.$$

OR

	C	C′	Total
A	0.25 · **0.50** 0.125	0.125	**0.25**
A′	0.075	0.75 · **0.90** 0.675	0.75
Total	**0.20**	0.80	1.00

b) Suppose CL1 lecture was good. What is the probability that AL1 lecture was also good?

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{0.125}{0.20} = \mathbf{\frac{5}{8}} = \mathbf{0.625}.$$

OR

$$P(A \mid C) = \frac{P(A) \times P(C \mid A)}{P(A) \times P(C \mid A) + P(A') \times P(C \mid A')} = \frac{0.25 \times 0.50}{0.25 \times 0.50 + 0.75 \times 0.10} = \frac{0.125}{0.20} = \mathbf{\frac{5}{8}}.$$

c) If at the conclusion of the CL1 lecture, the instructor is heard to mutter "what a rotten lecture," what is the probability that the AL1 lecture was also bad?

$$P(A' \mid C') = \frac{P(A' \cap C')}{P(C')} = \frac{0.675}{0.80} = \frac{\mathbf{27}}{\mathbf{32}} = \mathbf{0.84375}.$$

d) What is the probability that at least one lecture is good (i.e., either AL1 lecture is good or CL1 lecture is good, or both)?

$$P(A \cup C) = P(A) + P(C) - P(A \cap C) = 0.25 + 0.20 - 0.125 = \mathbf{0.325}.$$

OR

$$P(A \cup C) = P(A \cap C) + P(A' \cap C) + P(A \cap C') = 0.125 + 0.075 + 0.125 = \mathbf{0.325}.$$

OR

$$P(A \cup C) = 1 - P(A' \cap C') = 1 - 0.675 = \mathbf{0.325}.$$

2. **1.3-4 (c)** **1.4-4 (c)**

Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing a heart on the first draw and an ace on the second draw.

Hint: Note that a heart can be drawn by getting the ace of hearts or one of the other 12 hearts.

$$P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$$

$$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{48+3}{52 \cdot 51} = \frac{51}{52 \cdot 51} = \frac{\mathbf{1}}{\mathbf{52}}.$$

3. Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B.

a) **1.3-16** **1.4-18**

Compute the probability of then drawing a red chip from bowl B.

$$P(R_B) = P(R_A R_B) + P(W_A R_B)$$

$$= P(R_A) \times P(R_B \mid R_A) + P(W_A) \times P(R_B \mid W_A)$$

$$= \frac{3}{5} \cdot \frac{5}{8} + \frac{2}{5} \cdot \frac{4}{8} = \frac{15}{40} + \frac{8}{40} = \mathbf{\frac{23}{40}} = \mathbf{0.575}.$$

b) <u>If a red chip was drawn from bowl B</u>, what is the probability that a red chip had been drawn from bowl A?

$$P(R_A \mid R_B) = \frac{P(R_A R_B)}{P(R_B)} = \frac{\frac{15}{40}}{\frac{23}{40}} = \mathbf{\frac{15}{23}} \approx 0.652.$$

4. Consider two events, A and B, such that P(A) = 0.70, P(B) = 0.40 and P(A ∪ B) = 0.82. Are A and B independent?

$P(A \cup B) = P(A) + P(B) - P(A \cap B).$

$0.82 = 0.70 + 0.40 - P(A \cap B).$

$\Rightarrow \quad P(A \cap B) = 0.28.$

$P(A \cap B) = 0.28 = 0.70 \times 0.40 = P(A) \times P(B).$

$\Rightarrow$ A and B are **independent**.

5. At a hospital's emergency room, patients are classified and 20% of them are critical, 30% are serious, and 50% are stable. Of the critical ones, 30% die; of the serious, 10% die; and of the stable, 2% die.

a) ~ **1.5-5** ~ **1.6-5**

Given that a patient dies, what is the conditional probability that the patient was classified as critical? As serious? As stable?

$$\mathrm{P}(\text{patient dies}) = 0.20 \times 0.30 + 0.30 \times 0.10 + 0.50 \times 0.02$$
$$= 0.06 + 0.03 + 0.01 = 0.10.$$

$$\mathrm{P}(\text{critical} \mid \text{patient dies}) = \frac{0.06}{0.10} = \mathbf{0.60}.$$

$$\mathrm{P}(\text{serious} \mid \text{patient dies}) = \frac{0.03}{0.10} = \mathbf{0.30}.$$

$$\mathrm{P}(\text{stable} \mid \text{patient dies}) = \frac{0.01}{0.10} = \mathbf{0.10}.$$

b) Are events {a patient dies} and {a patient was classified as critical} independent? ***Justify your answer***.

$$\mathrm{P}(\text{patient dies} \cap \text{critical}) = 0.06 \neq 0.10 \times 0.20 = \mathrm{P}(\text{patient dies}) \times \mathrm{P}(\text{critical}).$$
$$\mathrm{P}(\text{patient dies} \mid \text{critical}) = 0.30 \neq 0.10 = \mathrm{P}(\text{patient dies}).$$
$$\mathrm{P}(\text{critical} \mid \text{patient dies}) = 0.60 \neq 0.20 = \mathrm{P}(\text{critical}).$$

{a patient dies} and {a patient was classified as critical} are **NOT independent**.

c) Are events {a patient dies} and {a patient was classified as serious} independent? ***Justify your answer***.

$$\mathrm{P}(\text{patient dies} \cap \text{serious}) = 0.03 = 0.10 \times 0.30 = \mathrm{P}(\text{patient dies}) \times \mathrm{P}(\text{serious}).$$
$$\mathrm{P}(\text{patient dies} \mid \text{serious}) = 0.10 = 0.10 = \mathrm{P}(\text{patient dies}).$$
$$\mathrm{P}(\text{serious} \mid \text{patient dies}) = 0.30 = 0.30 = \mathrm{P}(\text{serious}).$$

{a patient dies} and {a patient was classified as critical} are **independent**.

6. Does a monkey have a better chance of rearranging

A C C L L S U U   to spell   C A L C U L U S

or

A A B E G L R   to spell   A L G E B R A ?

A C C L L S U U $\rightarrow$ C A L C U L U S $\dfrac{8!}{1!\cdot 2!\cdot 2!\cdot 1!\cdot 2!} = 5040$ ways.

A A B E G L R $\rightarrow$ A L G E B R A $\dfrac{7!}{2!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = 2520$ ways.

Spelling A L G E B R A is twice as likely as spelling C A L C U L U S.

7. Three cards are drawn at random without replacement from a standard 52-card deck. What is the probability that either all three cards are face cards (king, queen, jack) or all three cards are red cards (♦, ♥) (or both)?

P(3 face $\cup$ 3 red) = P(3 face) + P(3 red) – P(3 face $\cap$ 3 red)

$$\frac{{}_{12}C_3 \cdot {}_{40}C_0}{{}_{52}C_3} + \frac{{}_{26}C_3 \cdot {}_{26}C_0}{{}_{52}C_3} - \frac{{}_{6}C_3 \cdot {}_{46}C_0}{{}_{52}C_3} \approx \mathbf{0.1267}.$$

OR

$$\frac{12}{52}\cdot\frac{11}{51}\cdot\frac{10}{50} + \frac{26}{52}\cdot\frac{25}{51}\cdot\frac{24}{50} - \frac{6}{52}\cdot\frac{5}{51}\cdot\frac{4}{50} \approx \mathbf{0.1267}.$$

8. A laboratory has 15 rats, 7 white, 6 gray and 2 brown. Suppose 5 rats will be selected at random and assigned to an experimental drug.

a) Find the probability that 2 white, 2 gray and 1 brown rats were selected.

$$\begin{array}{ccc} & 15 & \\ \swarrow & \downarrow & \searrow \\ 7 & 6 & 2 \\ \text{W} & \text{G} & \text{B} \\ \downarrow & \downarrow & \downarrow \\ 2 & 2 & 1 \end{array}$$

$$\frac{{}_7C_2 \cdot {}_6C_2 \cdot {}_2C_1}{{}_{15}C_5} = \frac{21 \cdot 15 \cdot 2}{3003} = \frac{630}{3003} \approx \mathbf{0.20979}.$$

b) Find the probability that 3 white and 2 gray rats were selected.

$$\begin{array}{ccc} & 15 & \\ \swarrow & \downarrow & \searrow \\ 7 & 6 & 2 \\ \text{W} & \text{G} & \text{B} \\ \downarrow & \downarrow & \downarrow \\ 3 & 2 & 0 \end{array}$$

$$\frac{{}_7C_3 \cdot {}_6C_2 \cdot {}_2C_0}{{}_{15}C_5} = \frac{35 \cdot 15 \cdot 1}{3003} = \frac{525}{3003} \approx \mathbf{0.174825}.$$

c) Find the probability that all five rats selected have the same color.

P(all five rats have the same color) = P(all five are white) + P(all five are gray)

$$= \frac{{}_7C_5 \cdot {}_6C_0 \cdot {}_2C_0}{{}_{15}C_5} + \frac{{}_7C_0 \cdot {}_6C_5 \cdot {}_2C_0}{{}_{15}C_5} = \frac{21 \cdot 1 \cdot 1}{3003} + \frac{1 \cdot 6 \cdot 1}{3003}$$

$$= \frac{27}{3003} \approx \mathbf{0.008991}.$$

OR

P(all five rats have the same color) = P(all five are white) + P(all five are gray)

$$= \left[\frac{7}{15} \cdot \frac{6}{14} \cdot \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{3}{11}\right] + \left[\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{4}{13} \cdot \frac{3}{12} \cdot \frac{2}{11}\right] = \frac{2520}{360360} + \frac{720}{360360}$$

$$= \frac{3240}{360360} \approx \mathbf{0.008991}.$$

d) Find the probability that all five rats selected have different color.

P(all five rats selected have different color) = **0**. (3 colors, 5 rats)

9. A bank has two emergency sources of power for its computers. There is a 95% chance that source 1 will operate during a total power failure, and an 80% chance that source 2 will operate. Assume the power sources are independent. What is the probability that at least one of them will operate during a total power failure?

$0.95 + 0.05 \times 0.80 = \mathbf{0.99}$ OR $1 - 0.05 \times 0.20 = \mathbf{0.99}$.

10. If a fair 6-sided die is rolled 6 times, what is the probability that each possible outcome (1, 2, 3, 4, 5, and 6) will occur exactly once?

$$\frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} = \frac{6!}{6^6} = \mathbf{\frac{5}{324}} \approx \mathbf{0.0154321}.$$

11. **1.3-7** **1.4-9**

An urn contains four colored balls: two orange and two blue. Two balls are selected at random without replacement, and you are told that at least one of them is orange. What is the probability that the other ball is also orange?

Hint 1: O O O B B O B B

Hint 2: $\mathrm{P}(\text{both orange} \mid \text{at least one orange})$.

Outcome	O O	O B	B O	B B
Probability	$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$	$\frac{2}{4} \times \frac{2}{3} = \frac{4}{12}$	$\frac{2}{4} \times \frac{2}{3} = \frac{4}{12}$	$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$

$$\mathrm{P}(\text{both orange} \mid \text{at least one orange}) = \frac{\mathrm{P}(\text{both orange} \cap \text{at least one orange})}{\mathrm{P}(\text{at least one orange})}$$

$$= \frac{\mathrm{P}(\{\mathrm{OO}\} \cap \{\mathrm{OO},\ \mathrm{OB},\ \mathrm{BO}\})}{\mathrm{P}(\{\mathrm{OO},\ \mathrm{OB},\ \mathrm{BO}\})} = \frac{\mathrm{P}(\{\mathrm{OO}\})}{\mathrm{P}(\{\mathrm{OO},\ \mathrm{OB},\ \mathrm{BO}\})} = \frac{2/12}{10/12} = \mathbf{\frac{1}{5}}.$$

12. A new flavor of toothpaste has been developed. It was tested by a group of 15 people. Nine of the group said they liked the new flavor, and the remaining 6 indicated they did not. Five of the 15 are selected at random to participate in an in-depth interview.

a) What is the probability that of those selected for the in-depth interview 4 liked the new flavor and 1 did not?

$$\frac{{}_9C_4 \cdot {}_6C_1}{{}_{15}C_5} \approx \mathbf{0.2517}.$$

b) What is the probability that of those selected for the in-depth interview at least 4 liked the new flavor?

$$\frac{{}_9C_4 \cdot {}_6C_1}{{}_{15}C_5} + \frac{{}_9C_5 \cdot {}_6C_0}{{}_{15}C_5} \approx \mathbf{0.2937}.$$

13. A company that manufactures video cameras produces a basic model and a deluxe model. Suppose that 25% of the cameras sold are the basic model. Of those buying the basic model, 20% purchase an extended warranty, whereas 40% of the deluxe model purchasers do so.

$P(B) = 0.25.$ $\quad P(W \mid B) = 0.20,$ $\quad P(W \mid D) = 0.40.$

a) What proportion of customers purchase an extended warranty?

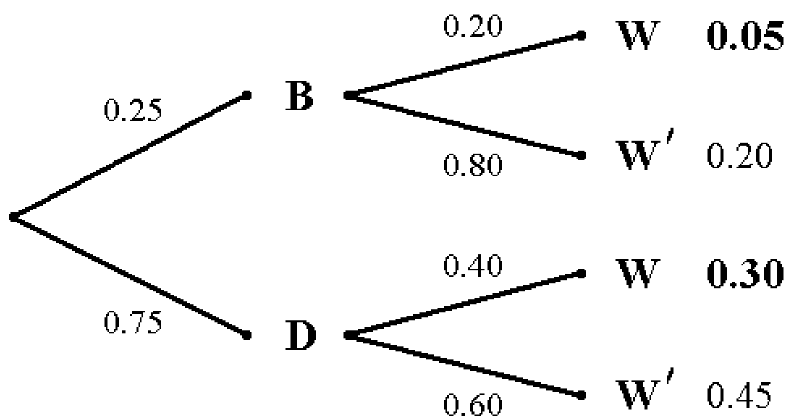
$$\begin{aligned} P(W) &= P(B) \cdot P(W \mid B) + P(D) \cdot P(W \mid D) \\ &= 0.25 \cdot 0.20 + 0.75 \cdot 0.40 = \mathbf{0.35}. \end{aligned}$$

(Law of Total Probability)

OR

	W	W'	
B	$0.25 \cdot \mathbf{0.20}$ 0.05	0.20	**0.25**
D	$0.75 \cdot \mathbf{0.40}$ 0.30	0.45	0.75
	0.35	0.65	1.00

OR



$P(W) = 0.05 + 0.30 = \mathbf{0.35}$.

b) What proportion of customers either purchase an extended warranty or purchase a deluxe model, or both?

$P(W \cup D) = P(W) + P(D) - P(W \cap D) = 0.35 + 0.75 - 0.30 = \mathbf{0.80}$.

OR

$P(W \cup D) = P(W \cap D) + P(W \cap B) + P(W' \cap D) = 0.30 + 0.05 + 0.45 = \mathbf{0.80}$.

OR

$P(W \cup D) = 1 - P(W' \cap B) = 1 - 0.20 = \mathbf{0.80}$.

c) If you learn that a randomly selected purchaser has an extended warranty, what is the probability that he/she has a deluxe model?

$$P(D \mid W) = \frac{P(D \cap W)}{P(W)} = \frac{0.30}{0.35} = \mathbf{\frac{6}{7}} \approx \mathbf{0.8571}.$$

d) Suppose a randomly selected purchaser does not have an extended warranty. What is the probability that he/she has a basic model?

$$P(B \mid W') = \frac{P(B \cap W')}{P(W')} = \frac{0.20}{0.65} = \mathbf{\frac{4}{13}} \approx \mathbf{0.3077}.$$