STAT 400
Exam I

Basic Discrete RVs

Fall 2017
Pracitce

1. Homer Simpson is going to Moe's Bar for some *Flaming Moe*'s. Let X denote the number of *Flaming Moe*'s that Homer Simpson will drink. Suppose X has the following probability distribution:

x	$f(x)$
0	0.1
1	0.2
2	0.3
3	0.3
4	

a) Find the probability $f(4)$ = P(X = 4).

b) Find the probability P(X ≥ 1).

c) Find the probability P(X ≥ 1 | X < 3).

d) Compute the expected value of X, E(X).

e) Compute the standard deviation of X, SD(X).

1. (continued)

Suppose each *Flaming Moe* costs $1.50, and there is a cover charge of $1.00 at the door. Let Y denote the amount of money Homer Simpson spends at the bar. Then $Y = 1.50 \cdot X + 1.00$.

f) Find the probability that Homer would spend over $5.00.

g) Find the expected amount of money that Homer Simpson would spend, E(Y).

h) Find the standard deviation for the amount of money that Homer Simpson would spend, SD(Y).

2. Each of three balls is randomly placed into one of three bowls. Let X denote the number of empty bowls.

a) Find the probability distribution of X.

"Hint": $f(0) = P(X = 0)$ and $f(2) = P(X = 2)$ are easier to find.

b) Find $E(X)$.

3. Suppose we roll a pair of fair 6-sided dice. Let Y denote the difference between the outcomes on the two dice. Construct the probability distribution of Y and compute its expected value.

4. Honest Harry introduced a new "savings opportunity" at *Honest Harry's Used Car Dealership*. A customer is offered the opportunity to roll four fair (balanced) 6-sided dice. If the customer rolls at least one "6", Honest Harry takes an extra $1,000 off the price of the car. However, if the customer does not have a "6", an extra $1,000 is added to the price of the car. What is the customer's expected savings from accepting this "savings opportunity"?

5. Upon examination of the claims records of its policy holders over a period of three years, an insurance company obtained the following probability distribution of X = number of claims in three years:

x	$f(x)$
0	0.35
1	0.30
2	0.20
3	0.10
4	0.05

a) Calculate the average number of claims per policy holder, $E(X)$.

b) Calculate the standard deviation of X, $SD(X)$.

6. **2.1-10** **2.1-10**

Suppose there are 3 defective items in a lot (collection) of 50 items. A sample of size 10 is taken at random and without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains

a) Exactly one defective item.

b) At most one defective item.

7. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f.

$$f(x) = \frac{5-x}{10}, \qquad x = 1,2,3,4.$$

a) Find $\mathrm{E}(\mathrm{X})$, the expected number of days the patient needs to be in the hospital.

b) **2.2-3** **2.2-5**

If the patient is to receive \$200 from the insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

"Hint": If the patient spends three days in the hospital, the payment is \$500.

8. **2.2-4** **2.2-6**

An insurance company sells an automobile policy with a deductible of one unit. Let X be the amount of the loss having pmf

$$f(x) = \begin{cases} 0.9, & x = 0, \\ \dfrac{c}{x}, & x = 1, 2, 3, 4, 5, 6. \end{cases}$$

where c is a constant. Determine c and the expected value of the amount the insurance company must pay.

9. Mr. George Room proposed to Ms. Brittany Ride with a diamond ring valued at $8,000. Ms. B. Ride thinks the probability that it will be lost or stolen in a given year is .006. She can buy an *Total* insurance policy for one year for $50 that will completely reimburse the loss of the ring if it gets lost or stolen. She can also buy *Regular* insurance policy for one year for $30 that will reimburse 75% of the cost of the ring if it gets lost or stolen.

a) Determine the payoff table.

	θ_1 ring is lost or stolen	θ_2 ring is fine
a_1 buy *Total*		
a_2 buy *Regular*		
a_3 do not buy insurance		

b) Compute the Expected Payoff for all three actions and determine the optimal action.

10. A credit card company examines its records to determine the length of time (in weeks) between the time a bill is sent and payment is received. The following table shows the results:

Number of Weeks	1	2	3	4	5
Proportion	0.15	0.30	0.25		0.10

Let X denote the number of weeks between the time a bill is sent and payment is received.

a) Find the missing proportion.

b) Find $P(X > 3)$.

c) Find $P(2 \le X \le 4)$.

d) Find the expected (average) number of weeks between the time a bill is sent and payment is received, $E(X)$.

e) Find $Var(X)$.

f) Find $SD(X)$.

Answers:

1. Homer Simpson is going to Moe's Bar for some *Flaming Moe*'s. Let X denote the number of *Flaming Moe*'s that Homer Simpson will drink. Suppose X has the following probability distribution:

x	$f(x)$	$x\,f(x)$	$x^2\,f(x)$
0	0.1	0.0	0.0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.3	0.9	2.7
4	*0.1*	0.4	1.6
	1.0	2.1	5.7

a) Find the probability $f(4)$ = P(X = 4).

$f(4) = 1 - [\,0.1 + 0.2 + 0.3 + 0.3\,] =$ **0.10**.

b) Find the probability P(X ≥ 1).

P(X ≥ 1) = **0.90**.

c) Find the probability P(X ≥ 1 | X < 3).

$$\text{P}(\text{X} \ge 1 \mid \text{X} < 3) = \frac{\text{P}(\text{X} \ge 1 \cap \text{X} < 3)}{\text{P}(\text{X} < 3)} = \frac{0.5}{0.6} = \mathbf{0.8333}.$$

d) Compute the expected value of X, E(X).

$$\text{E}(\text{X}) = \sum_{\text{all x}} \text{x} \cdot f(\text{x}) = \mathbf{2.1}.$$

e) Compute the standard deviation of X, SD(X).

$$\text{Var}(\text{X}) = \sum_{\text{all x}} \text{x}^2 \cdot f(\text{x}) - [\text{E}(\text{X})]^2 = 5.7 - (2.1)^2 = 1.29.$$

$$\text{SD}(\text{X}) = \sqrt{1.29} = \mathbf{1.1358}.$$

1. (continued)

Suppose each *Flaming Moe* costs \$1.50, and there is a cover charge of \$1.00 at the door. Let Y denote the amount of money Homer Simpson spends at the bar. Then $Y = 1.50 \cdot X + 1.00$.

f) Find the probability that Homer would spend over \$5.00.

x	y	$f(x) = f(y)$
0	\$1.00	0.10
1	\$2.50	0.20
2	\$4.00	0.30
3	\$5.50	0.30
4	\$7.00	0.10
		1.00

$P(Y > \$5.00) = P(X \geq 3) =$ **0.40**.

g) Find the expected amount of money that Homer Simpson would spend, E(Y).

$\mu_Y = E(Y) = 1.50 \cdot E(X) + 1.00 =$ **\$4.15**.

(On average, Homer drinks 2.1 *Flaming Moe*'s per visit, his expected payment for the drinks is \$3.15. His expected total payment is \$4.15 since he has to pay \$1.00 for the over charge.)

OR

x	y	$f(x) = f(y)$	$y \cdot f(y)$
0	\$1.00	0.10	0.10
1	\$2.50	0.20	0.50
2	\$4.00	0.30	1.20
3	\$5.50	0.30	1.65
4	\$7.00	0.10	0.70
		1.00	**4.15**

$\mu_Y = E(Y) = \sum_{\text{all } y} y \cdot f(y) =$ **\$4.15**.

h) Find the standard deviation for the amount of money that Homer Simpson would spend, SD(Y).

σ_Y = SD(Y) = $|\,1.50\,|$ · SD(X) = **$1.7037**.

OR

x	y	$f(x) = f(y)$	$(y - \mu_Y)^2 \cdot f(y)$	$y^2 \cdot f(y)$
0	$1.00	0.10	0.99225	0.100
1	$2.50	0.20	0.54450	1.250
2	$4.00	0.30	0.00675	4.800
3	$5.50	0.30	0.54675	9.075
4	$7.00	0.10	0.81225	4.900
		1.00	**2.9025**	20.125

σ_Y^2 = Var(Y) = $\sum_{\text{all } y} (y - \mu_Y)^2 \cdot f(y)$ = **2.9025**.

OR

σ_Y^2 = Var(Y) = $\sum_{\text{all } y} y^2 \cdot f(y) - [\,E(Y)\,]^2$ = 20.125 – $(4.15)^2$

= 20.125 – 17.2225 = **2.9025**.

σ_Y = SD(Y) = $\sqrt{\mathbf{2.9025}}$ = **$1.7037**.

2. Each of three balls is randomly placed into one of three bowls. Let X denote the number of empty bowls.

a) Find the probability distribution of X.

"Hint": $f(0) = \mathrm{P}(\mathrm{X} = 0)$ and $f(2) = \mathrm{P}(\mathrm{X} = 2)$ are easier to find.

$$f(0) = \mathrm{P}(\mathrm{X} = 0) \;=\; \mathrm{P}(\text{no empty bowls}) \;=\; \mathrm{P}(\text{one ball in each bowl}) \;=\; \frac{3}{3} \times \frac{2}{3} \times \frac{1}{3} \;=\; \frac{2}{9}.$$

$$f(2) = \mathrm{P}(\mathrm{X} = 2) \;=\; \mathrm{P}(\text{two empty bowls}) \;=\; \mathrm{P}(\text{all balls in one bowl}) \;=\; \frac{3}{3} \times \frac{1}{3} \times \frac{1}{3} \;=\; \frac{1}{9}.$$

$$f(1) = \mathrm{P}(\mathrm{X} = 1) \;=\; 1 - f(0) - f(2) \;=\; 1 - \frac{2}{9} - \frac{1}{9} \;=\; \frac{6}{9}.$$

OR

1 2 3				1 2 3				1 2 3
1 2	3		3	1 2		3		1 2
1 2		3		1 2	3		3	1 2
1 3	2		2	1 3		2		1 3
1 3		2		1 3	2		2	1 3
1	2 3		2 3	1		2 3		1
1		2 3		1	2 3		2 3	1
1	2	3	2	1	3	2	3	1
1	3	2	3	1	2	3	2	1

b) Find $\mathrm{E}(\mathrm{X})$.

$$\mathrm{E}(\mathrm{X}) \;=\; \sum_{\text{all } x} x \cdot f(x) \;=\; 0 \cdot \frac{2}{9} + 1 \cdot \frac{6}{9} + 2 \cdot \frac{1}{9} \;=\; \boldsymbol{\frac{8}{9}}$$

3. Suppose we roll a pair of fair 6-sided dice. Let Y denote the difference between the outcomes on the two dice. Construct the probability distribution of Y and compute its expected value.

(1 , 1)	(1 , 2)	(1 , 3)	(1 , 4)	(1 , 5)	(1 , 6)
(2 , 1)	(2 , 2)	(2 , 3)	(2 , 4)	(2 , 5)	(2 , 6)
(3 , 1)	(3 , 2)	(3 , 3)	(3 , 4)	(3 , 5)	(3 , 6)
(4 , 1)	(4 , 2)	(4 , 3)	(4 , 4)	(4 , 5)	(4 , 6)
(5 , 1)	(5 , 2)	(5 , 3)	(5 , 4)	(5 , 5)	(5 , 6)
(6 , 1)	(6 , 2)	(6 , 3)	(6 , 4)	(6 , 5)	(6 , 6)

y	$f(y)$
0	$^{6}/_{36}$
1	$^{10}/_{36}$
2	$^{8}/_{36}$
3	$^{6}/_{36}$
4	$^{4}/_{36}$
5	$^{2}/_{36}$

$y \times f(y)$
0
$^{10}/_{36}$
$^{16}/_{36}$
$^{18}/_{36}$
$^{16}/_{36}$
$^{10}/_{36}$

$$\mathrm{E}(\mathrm{Y}) = {}^{70}/_{36} = {}^{35}/_{18} \approx 1.944444.$$

4. Honest Harry introduced a new "savings opportunity" at *Honest Harry's Used Car Dealership*. A customer is offered the opportunity to roll four fair (balanced) 6-sided dice. If the customer rolls at least one "6", Honest Harry takes an extra $1,000 off the price of the car. However, if the customer does not have a "6", an extra $1,000 is added to the price of the car. What is the customer's expected savings from accepting this "savings opportunity"?

P(no "6") = $\left(\frac{5}{6}\right)\cdot\left(\frac{5}{6}\right)\cdot\left(\frac{5}{6}\right)\cdot\left(\frac{5}{6}\right)=\frac{625}{1296}\approx 0.48225.$

P(at least one "6") = 1 – P(no "6") = $1-\frac{625}{1296}=\frac{671}{1296}\approx 0.51775.$

Let X denote the amount saved by the customer. Then X has the following probability distribution:

x	$p(x)$	$x \cdot p(x)$
$-1{,}000$	$\frac{625}{1296}$	-482.25
$1{,}000$	$\frac{671}{1296}$	517.75
		35.50

E(X) ≈ **$35.50**.

5. Upon examination of the claims records of its policy holders over a period of three years, an insurance company obtained the following probability distribution of X = number of claims in three years:

x	$f(x)$	$x \cdot f(x)$	$x^2 \cdot f(x)$
0	0.35	0.0	0.0
1	0.30	0.3	0.3
2	0.20	0.4	0.8
3	0.10	0.3	0.9
4	0.05	0.2	0.8
		1.2	2.8

a) Calculate the average number of claims per policy holder, E(X).

E(X) = **1.2**.

b) Calculate the standard deviation of X, SD(X).

Var(X) = 2.8 – 1.2 ^{2} = 1.36. SD(X) ≈ **1.1662**.

6. **2.1-10** **2.1-10**

Suppose there are 3 defective items in a lot (collection) of 50 items. A sample of size 10 is taken at random and without replacement. Let X denote the number of defective items in the sample. Find the probability that the sample contains

a) Exactly one defective item.

$$\frac{\binom{3}{1}\cdot\binom{47}{9}}{\binom{50}{10}} = \frac{\frac{3!}{1!\cdot 2!}\cdot\frac{47!}{9!\cdot 38!}}{\frac{50!}{10!\cdot 40!}} = \frac{3\cdot 10\cdot 40\cdot 39}{50\cdot 49\cdot 48} = \frac{39}{98} \approx 0.39796.$$

b) At most one defective item.

$$\frac{\binom{3}{0}\cdot\binom{47}{10}}{\binom{50}{10}} + \frac{\binom{3}{1}\cdot\binom{47}{9}}{\binom{50}{10}} = \frac{1\cdot\frac{47!}{10!\cdot 37!}}{\frac{50!}{10!\cdot 40!}} + \frac{39}{98} = \frac{40\cdot 39\cdot 38}{50\cdot 49\cdot 48} + \frac{39}{98}$$

$$= \frac{247}{490} + \frac{195}{490} = \frac{221}{245} \approx 0.90204.$$

7. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f.

$$f(x) = \frac{5-x}{10}, \qquad x = 1,2,3,4.$$

a) Find $\mathrm{E}(\mathrm{X})$, the expected number of days the patient needs to be in the hospital.

x	$f(x)$	$x \cdot f(x)$	
1	0.40	0.40	
2	0.30	0.60	
3	0.20	0.60	
4	0.10	0.40	
		2.00	$= \mathrm{E}(\mathrm{X})$

b) **2.2-3** **2.2-5**

If the patient is to receive \$200 from the insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

"Hint": If the patient spends three days in the hospital, the payment is \$500.

x	payment	$f(x)$	payment $\cdot f(x)$	
1	\$200	0.40	80	
2	\$400	0.30	120	
3	\$500	0.20	100	
4	\$600	0.10	60	
			\$360	$= \mathrm{E}(\text{payment})$

8. **2.2-4** **2.2-6**

An insurance company sells an automobile policy with a deductible of one unit. Let X be the amount of the loss having pmf

$$f(x) = \begin{cases} 0.9, & x = 0, \\ \dfrac{c}{x}, & x = 1, 2, 3, 4, 5, 6. \end{cases}$$

where c is a constant. Determine c and the expected value of the amount the insurance company must pay.

(In an insurance policy, the deductible is the portion of any claim that is not covered by the insurance provider.)

Must have $$1 = 0.9 + \frac{c}{1} + \frac{c}{2} + \frac{c}{3} + \frac{c}{4} + \frac{c}{5} + \frac{c}{6} = 0.9 + \frac{147 \cdot c}{60}.$$

$$\Rightarrow \quad c = \frac{6}{147} = \frac{2}{49}.$$

$$\mathrm{E}(\text{Payment}) = 1 \cdot \frac{c}{2} + 2 \cdot \frac{c}{3} + 3 \cdot \frac{c}{4} + 4 \cdot \frac{c}{5} + 5 \cdot \frac{c}{6} = \frac{213}{60} \cdot c = \mathbf{\frac{71}{490}}.$$

9. Mr. George Room proposed to Ms. Brittany Ride with a diamond ring valued at $8,000. Ms. B. Ride thinks the probability that it will be lost or stolen in a given year is .006. She can buy an *Total* insurance policy for one year for $50 that will completely reimburse the loss of the ring if it gets lost or stolen. She can also buy *Regular* insurance policy for one year for $30 that will reimburse 75% of the cost of the ring if it gets lost or stolen.

a) Determine the payoff table.

	θ_1 ring is lost or stolen	θ_2 ring is fine
a_1 buy *Total*	–$50 –$8,000 +$8,000 **–$50**	–$50 **–$50**
a_2 buy *Regular*	–$30 –$8,000 +$6,000 **–$2,030**	–$30 **–$30**
a_3 do not buy insurance	–$8,000 **–$8,000**	**$0**

b) Compute the Expected Payoff for all three actions and determine the optimal action.

EP(a_1) = **–$50**.

EP(a_2) = (–2,030) × 0.006 + (–30) × 0.994 = **–$42**.

EP(a_3) = (–8,000) × 0.006 + (0) × 0.994 = **–$48**.

Optimal action = $\mathbf{a_2}$ (buy *Regular*).

10. a) **0.20**. b) **0.30**.

c) **0.75**. d) **2.8**.

e) $9.3 - 2.8^2$ = **1.46**. f) **1.2083**.