More Interval and Testing Practice!

1. ***Taxes'R'Us Block*** is a national chain that prepares income tax returns for individuals. The president of the company wants an average number of 5 customers per clerk per day. On April 15, a random sample of 64 clerks shows the average number of customers per clerk at 4.75. Assume that the overall standard deviation of 1.15.

a) Test whether the average number of customers per clerk on April 15 was 5 or it was different. Use $\alpha = 0.05$.

b) Suppose the actual overall average number of customers per clerk on April 15 was 4.8. Was the decision you made in part (a) correct? If not, what type of error has been made?

c) What is the p-value of the test in part (a) ?

d) **Using the p-value obtained in part (c)**, would you accept or reject the null hypothesis from part (a) at $\alpha = 0.07$?

2. In its advertising, ***Taxes'R'Us Block*** claims that the overall average bill for its services does not exceed \$75. A consumer agency obtains a random sample of 20 tax returns prepared by the company and finds the average preparation bill for those 20 to be \$79.82 with a sample standard deviation of \$10.30. (Assume the amounts of the service fees are normally distributed.)

a) We would like to test the claim made in the ad. Perform the appropriate test at a 10% significance level.

b) What is the p-value of the test in part (a) ?

c) Was the decision you made in part (a) correct? If not, what type of error has been made?

d) Construct a 95% confidence interval for the average bill for services at this firm.

3. ***Taxes'R'Us Block*** is currently under an IRS investigation for possible underreporting of income by its customers on federal income tax returns.

a) A random sample of 225 ***Taxes'R'Us Block***'s customers is obtained, of which 72 underreported their income. Construct a 90% confidence interval for the proportion of all ***Taxes'R'Us Block***'s customers who underreport their income.

b) IRS claims that at least 35% of all ***Taxes'R'Us Block***'s customers underreported their income. Perform the appropriate test using a 5% level of significance.

c) What is the p-value of the test in part (b) ?

d) Find the minimum sample size required for estimating the proportion of all ***Taxes'R'Us Block***'s customers who underreport their income to within 3% with 90% confidence, if it is known that this proportion is between 0.25 and 0.35.

4. In its advertising, ***Taxes'R'Us Block*** claims that the average tax return for a family with two children is at least \$700. A random sample of 81 tax returns shows a sample mean of \$674 and a sample standard deviation of \$117.

a) Perform the appropriate test at a 1% level of significance.

b) What is the p-value of the test in part (a) ?

c) **Using the p-value obtained in part (b)**, would you accept or reject the null hypothesis from part (a) at $\alpha = 0.04$?

5. Suppose the time it takes to prepare a tax return at ***Taxes'R'Us Block*** is normally distributed with mean 27 minutes and standard deviation 5 minutes.

a) What proportion of tax returns take more than 30 minutes to prepare?

b) What is the probability that the average preparation time for a random sample of 16 customers will be greater than 29 minutes?

Answers:

1. $\overline{X} = 4.75$ $\sigma = 1.15$ $n = 64$

a) Claims : $\mu = 5$ (average number of customers per clerk was 5) and
$\mu \neq 5$ (it was different).

$H_0 : \mathbf{\mu = 5}$ vs. $H_1 : \mathbf{\mu \neq 5}$. Two - tail.

Test Statistic: σ is known. $T = \dfrac{\overline{X} - \mu_0}{s / \sqrt{n}} = \dfrac{44{,}400 - 45{,}000}{2{,}000 / \sqrt{25}} = \mathbf{-1.74}$.

Rejection Region: $Z < t_{\alpha/2}$ or $Z > t_{\alpha/2}$

$\alpha = 0.05$ $\alpha/2 = \mathbf{0.025}$. $z_{0.025} = \mathbf{1.960}$.

The value of the test statistic **does not** fall into the Rejection Region.

Accept H_0 (Do not reject H_0) at $\alpha = 0.05$.

OR

σ is known. The confidence interval : $\overline{X} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$.

$\alpha = 0.05$ 95% conf. level. $t_{\alpha/2} = \mathbf{1.960}$.

$7.9 \pm 2.131 \dfrac{0.21}{\sqrt{16}}$ $\mathbf{4.75 \pm 0.28}$ **(4.47 ; 5.03)**

Since **5** is covered by the confidence interval,

Accept H_0 (Do not reject H_0) at $\alpha = 0.05$.

b) Given : $\mu = \mathbf{4.8}$.

$\mu = \mathbf{4.8}$ makes H_0 false. Yet, in part (a), H_0 was not rejected.

	H_0 true	**H_0 false**
Do not Reject H_0	☺	Type II Error
Reject H_0	Type I Error	☺

Therefore, **Type II Error** was made.

c) p - value = $2 \cdot P(Z < -1.74) = 2 \cdot \Phi(-1.74) = 2 \cdot 0.0409 =$ **0.0818**.

d) p - value > α $\Rightarrow$ Accept H_0 p - value < α $\Rightarrow$ Reject H_0

Since $0.0818 > 0.07$, **Accept H_0 (Do not reject H_0) at $\alpha = 0.07$**.

2. $\overline{X} = 79.82$ $s = 10.30$ $n = 20$

a) Claim : $\mu \le 75$ (overall average bill does not exceed \$75).

$H_0 : \boldsymbol{\mu \le 75}$ vs. $H_1 : \boldsymbol{\mu > 75}$. Right - tail.

Test Statistic: σ is unknown. $T = \dfrac{\overline{X} - \mu_0}{s / \sqrt{n}} = \dfrac{44{,}400 - 45{,}000}{2{,}000 / \sqrt{25}} = \mathbf{2.093}$.

Rejection Region: $T > t_\alpha$

number of degrees of freedom = $n - 1 = 20 - 1 =$ **19**.

$\alpha = 0.10$. $t_{0.100} =$ **1.328**.

The value of the test statistic **does** fall into the Rejection Region.

Reject H_0 at $\alpha = 0.10$.

b) p - value = area to the right of $T = 2.093$ (19 degrees of freedom) = **0.025**.

c) Since we do not know the value of μ (the overall average preparation bill),

we **do not know** whether the decision in part (a) is correct or not.

	H_0 true	**H_0 false**
Do not Reject H_0	☺	Type II Error
Reject H_0	Type I Error	☺

If H_0 is FALSE, then the decision is correct.

If H_0 is TRUE, then the Type I Error is made.

d) σ is unknown. The confidence interval : $\overline{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$.

95% conf. level number of degrees of freedom = $n - 1 = 20 - 1 = \mathbf{19}$.

$t_{0.025} = \mathbf{2.093}$.

$7.9 \pm 2.131 \frac{0.21}{\sqrt{16}}$ **79.82 ± 4.82** **(75.00 ; 84.64)**

3. X = 72. $n = 225$. $\hat{p} = \frac{X}{n} = \frac{165}{500} = \mathbf{0.32}$.

a) The confidence interval : $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$.

90% conf. level $z_{0.05} = \mathbf{1.645}$.

$0.33 \pm 1.75 \sqrt{\frac{(0.33)(0.67)}{500}}$ **0.32 ± 0.051** **(0.269 , 0.371)**

b) Claim : $p \geq 0.35$ (at least 35% of all ***Taxes'R'Us Block***'s customers …).

$H_0 : \boldsymbol{p \geq 0.35}$ vs. $H_1 : \boldsymbol{p < 0.35}$. Left - tail. $\alpha = 0.05$.

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1 - p_0)}{n}}} = \frac{0.235 - 0.20}{\sqrt{\frac{0.20 \cdot 0.80}{400}}} = \mathbf{-0.94}.$$

Rejection Region: $Z < -z_\alpha$ $z_{0.05} = \mathbf{1.645}$.

The value of the test statistic **does not** fall into the Rejection Region.

Accept H_0 (Do not reject H_0) at $\alpha = 0.10$.

c) p - value = P($Z < -0.94$) = $\Phi(-0.94)$ = **0.1736**.

d) It is known that this proportion is between 0.25 and 0.35, use $p = \mathbf{0.35}$ (the closest to 0.50 possible value).

$\varepsilon = 0.03$ (3%). 90% conf. level $z_{0.05} = \mathbf{1.645}$.

$$n = \frac{(z_{\alpha/2})^2 \cdot \hat{p} \cdot (1 - \hat{p})}{B^2} = \frac{(1.75)^2 \cdot 0.40 \cdot 0.60}{0.03^2} = \mathbf{684.023}.$$

Round **up**. $n = \mathbf{685}$.

4. $\overline{X} = 674$ $\quad$ $s = 117$ $\quad$ $n = 81$

a) Claims : $\mu \ge 700$ (average tax return ia at least \$700)

$H_0 : \boldsymbol{\mu \ge 700}$ vs. $H_1 : \boldsymbol{\mu < 700}$. Left - tail.

Test Statistic: σ is unknown.

$$T = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{44{,}400 - 45{,}000}{2{,}000/\sqrt{25}} = \mathbf{-2.00}.$$

Rejection Region: $T < -t_\alpha$

$\alpha = 0.01.$ number of degrees of freedom $= n - 1 = 81 - 1 = \mathbf{80}$.

$n = 81$ – large. Can use Z. $T < -z_\alpha$ $\quad$ $z_{0.01} = \mathbf{2.326}$.

The value of the test statistic **does not** fall into the Rejection Region.

Accept H_0 (Do not reject H_0) at $\alpha = 0.01$.

b) p - value = area to the left of $T = -2.00$ (80 degrees of freedom).

$n = 81$ – large. Can use Z. $P(Z \le -2.00) = \Phi(-2.00) = \mathbf{0.0228}$.

c) p - value $> \alpha$ $\Rightarrow$ Accept H_0 $\qquad$ p - value $< \alpha$ $\Rightarrow$ Reject H_0

Since p - value < 0.04, **Reject H_0 at $\alpha = 0.04$.**

5. $\mu = 27.$ $\quad$ $\sigma = 5.$

a) $$P(X > 30) = P\left(Z > \frac{30-27}{5}\right) = P(Z > 0.6) = 1 - \Phi(0.60) = 1 - 0.7257 = \mathbf{0.2743}.$$

b) $n = 16.$ Central Limit Theorem (Case 2):

$$P(\overline{X} > 29) = P\left(Z > \frac{29-27}{5/\sqrt{16}}\right) = P(Z > 1.6) = 1 - \Phi(1.60) = 1 - 0.9452 = \mathbf{0.0548}.$$