STAT 400 Fall 2017

Even More Interval and Testing Practice!!

1. Suppose the IQs of students at Anytown State University are normally distributed with standard deviation 15 and unknown mean.

a) Suppose a random sample of 64 students is obtained. Find the probability that the average IQ of the students in the sample will be within 3 points of the overall mean.

b) A sample of 64 students had a sample mean IQ of 115. Construct a 95% confidence interval for the overall mean IQ of students at Anytown State University.

c) What is the minimum sample size required if we want to estimate the overall mean IQ of students at Anytown State University to within 3 points with 95% confidence?

d) A university brochure boasts that the average IQ of students at Anytown State University is at least 120. A sample of 64 students had a sample mean IQ of 115. Perform the appropriate test at a 5% significance level.

e) Find the p-value of the test in part (d).

f) Suppose that only 20% of the students at Anytown State University have the IQ above 130. Find the overall average IQ of the students.

"Hint": From now on, you have μ.

g) (Type I Error, Type II Error, correct decision) was made in part (d).

h) Find the probability that the sample average IQ will be 115 or higher for a random sample of 64 students.

i) Only students in the top 33% are allowed to join the science club. What is the minimum IQ required to be able to join the science club?

j) What proportion of the students have IQ of 127 or above?

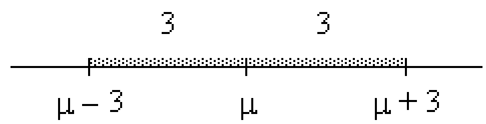
k) Find the probability that exactly 2 out of 6 randomly and independently selected students have IQ of 127 or above.

Answers:

1. Suppose the IQs of students at Anytown State University are normally distributed with standard deviation 15 and unknown mean.

a) Suppose a random sample of 64 students is obtained. Find the probability that the average IQ of the students in the sample will be within 3 points of the overall mean.

$\sigma = 15.$ $\quad \mu = ?$ $\quad n = 64.$



Need $P(\mu - 3 \le \overline{X} \le \mu + 3) = ?$

$n = 64$ – large (plus the distribution we sample from is normal).

Central Limit Theorem: $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} = Z.$

$$P(\mu - 3 \le \overline{X} \le \mu + 3) = P\left(\frac{(\mu - 3) - \mu}{15/\sqrt{64}} \le Z \le \frac{(\mu + 3) - \mu}{15/\sqrt{64}} \right)$$

$$= P(-1.60 \le Z \le 1.60) = \Phi(1.60) - \Phi(-1.60)$$

$$= 0.9452 - 0.0548 = \mathbf{0.8904}.$$

b) A sample of 64 students had a sample mean IQ of 115. Construct a 95% confidence interval for the overall mean IQ of students at Anytown State University.

$\overline{X} = 115$ $\quad \sigma = 15$ $\quad n = 64$

σ is known. The confidence interval : $\overline{X} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$.

$\alpha = 0.05$ $\quad a/2 = 0.025.$ $\quad z_{\alpha/2} = 1.96.$

$115 \pm 1.96 \cdot \dfrac{15}{\sqrt{64}}$ $\quad$ **115 ± 3.675** $\quad$ **(111.325 ; 118.675)**

c) What is the minimum sample size required if we want to estimate the overall mean IQ of students at Anytown State University to within 3 points with 95% confidence?

$\varepsilon = 3.$ $\sigma = 15.$ $\alpha = 0.05.$ $a/2 = 0.025.$ $z_{a/2} = 1.96.$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right)^2 = \left(\frac{1.96 \cdot 15}{3} \right)^2 = \mathbf{96.04}. \qquad \text{Round } \textbf{up}. \qquad n = \mathbf{97}.$$

d) A university brochure boasts that the average IQ of students at Anytown State University is at least 120. A sample of 64 students had a sample mean IQ of 115. Perform the appropriate test at a 5% significance level.

$H_0 : \mu \geq 120$ vs $H_1 : \mu < 120.$ Left – tailed.

$\overline{X} = 115.$ $\sigma = 15.$ $n = 64.$ $\alpha = 0.05.$

σ is known.

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{115 - 120}{15/\sqrt{64}} = \mathbf{-2.67}.$$

Rejection Region: $Z < -z_{\alpha}.$ $-z_{0.05} = -1.645.$

The value of the test statistic is in the Rejection Region.

Reject H_0 at $\alpha = 0.05$.

OR

P-value = (Area to the left of Z = –2.67) = P(Z < –2.67) = **0.0038**.

P-value < α = 0.05. **Reject H_0 at $\alpha = 0.05$.**

e) Find the p-value of the test in part (d).

P-value = (Area to the left of Z = –2.67) = P(Z < –2.67) = **0.0038**.

f) Suppose that only 20% of the students at Anytown State University have the IQ above 130. Find the overall average IQ of the students.

Know $P(X > 130) = 0.20$.

① Find z such that $P(Z > z) = 0.20$.

$\Phi(z) = 0.80.$ $z = 0.84.$

② $x = \mu + \sigma \cdot z.$ $130 = \mu + 15 \cdot (0.84).$ $\mu = \mathbf{117.4}.$

"Hint": From now on, you have μ.

g) (Type I Error, Type II Error, correct decision) was made in part (d).

$\mu = 117.4$ makes $H_0 : \mu \geq 120$ false. In part (d), H_0 was rejected.

	H_0 true	H_0 false
Do NOT reject H_0	☺	Type II Error
Reject H_0	Type I Error	☺

Therefore, a **correct decision** was made.

h) Find the probability that the sample average IQ will be 115 or higher for a random sample of 64 students.

Need $P(\overline{X} \geq 115) = ?$

$n = 64$ – large (plus the distribution we sample from is normal).

Central Limit Theorem: $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} = Z.$

$$P(\overline{X} \ge 115) = P\left(Z \ge \frac{115 - 117.4}{15/\sqrt{64}}\right) = P(Z \ge -1.28) = 1 - \Phi(-1.28)$$

$$= 1 - 0.1003 = \mathbf{0.8997}.$$

i) Only students in the top 33% are allowed to join the science club. What is the minimum IQ required to be able to join the science club?

Need x = ? such that $P(X > x) = 0.33$.

① Find z such that $P(Z > z) = 0.33$.

$\Phi(z) = 0.67.$ $z = 0.44.$

② $x = \mu + \sigma \cdot z.$ $x = 117.4 + 15 \cdot (0.44) = \mathbf{124}.$

j) What proportion of the students have IQ of 127 or above?

$$P(X \ge 127) = P\left(Z \ge \frac{127 - 117.4}{15}\right) = P(Z \ge 0.64) = 1 - \Phi(0.64)$$

$$= 1 - 0.7389 = \mathbf{0.2611}.$$

k) Find the probability that exactly 2 out of 6 randomly and independently selected students have IQ of 127 or above.

Let Y = number of students (out of the 6 selected) who have IQ of 127 or above.
Then Y has Binomial distribution, $n = 6$, $p = 0.2611$ (see part (j)).

$$P(Y = 2) = \binom{6}{2} 0.2611^2 \, 0.7389^4 = \mathbf{0.3048}.$$