STAT 400 | **Discussion 13** | Spring 2018

# Discussion 13

**1.** In a random sample of 200 students who took Exam P in 2017, 74 successfully passed the exam. ( Exam P [ Probability ] is the first Actuarial Science exam. )

a) Construct a 95% confidence interval for the overall proportion of students who successfully passed Exam P in 2017.

b) Find the minimum sample size required for estimating the overall proportion of students who passed Exam P in 2017 to within 4% with 95% confidence if it is known that this proportion is between 30% and 40%.

c) In 2016, the success rate on Exam P was 41%. At a 10% level of significance, test whether the success rate on Exam P in 2017 has changed from 2016. Find the p-value of this test.

**1.** (continued)

Suppose also that in a random sample of 150 students who took Exam FM in 2017, 45 successfully passed the exam. ( Exam FM [ Financial Mathematics ] is the second Actuarial Science exam. )

d) Construct a 95% confidence interval for the difference between the overall proportions of students who passed Exam P and Exam FM in 2017.

e) At a 10% level of significance, test whether Exam P had a higher success rate than Exam FM in 2017. Find the p-value of this test.

**2.** A random sample of 9 adult white rhinos had the sample mean weight of 5,100 pounds and the sample standard deviation of 450 pounds. A random sample of 16 adult hippos had the sample mean weight of 3,300 pounds and the sample standard deviation of 400 pounds. Assume that the two populations are approximately normally distributed.

a) Construct a 95% confidence interval for the difference between their overall average weights of adult white rhinos and adult hippos.

i) Assume that the overall standard deviations are equal.

ii) Do NOT assume that the overall standard deviations are equal. Use Welch's T.

iii) Do NOT assume that the overall standard deviations are equal. Use conservative approach.

b) Test $H_0: \mu_R - \mu_H = 1{,}500$ vs. $H_1: \mu_R - \mu_H > 1{,}500$ at $\alpha = 0.05$.

i) Assume that the overall standard deviations are equal.

ii) Do NOT assume that the overall standard deviations are equal. Use Welch's T.

**3.** In a random sample of eight 8-to-12-year-old children, the sample mean time playing video games was 48.8 hours per month with sample standard deviation 3.5 hours. In a random sample of ten 13-to-18-year-olds, the sample mean time playing video games was 54.3 hours per month with sample standard deviation 5.3 hours. Assume that the two populations are normally distributed.

a) Assume that the two population variances are equal. Construct a 95% confidence interval for the difference between are average time 8-to-12-year-olds and 13-to-18-year-olds play video games per month.

b) Since the larger sample variance is more than twice as big as the smaller one, the assumption of equal variances is questionable here. Construct a 95% confidence interval for the difference between are average time 8-to-12-year-olds and 13-to-18-year-olds play video games per month without assuming that the two population variances are equal. Use Welch's T.

**4.** A researcher wishes to determine whether the starting salaries of high-school math teachers in private schools are higher than those of high-school math teachers in public schools. She selects a sample of new math teachers from each type of school and calculates the sample means and sample standard deviations of their salaries.

| | Private | Public |
|---|---|---|
| sample size | 10 | 7 |
| sample mean | \$36,800 | \$36,300 |
| sample standard deviation | \$600 | \$546 |

Assume that the populations are normally distributed and the population variances are equal.

a) Construct a 95% confidence interval for the difference in average starting salaries of high-school math teachers in private and public schools.

b) Perform the appropriate test at a 1% level of significance. That is, test $H_0 : \mu_{\text{Private}} = \mu_{\text{Public}}$ vs. $H_1 : \mu_{\text{Private}} > \mu_{\text{Public}}$.

c) Find the p-value of the test in part (b).

d) Suppose instead we wish to test $H_0 : \mu_{\text{Private}} = \mu_{\text{Public}}$ vs. $H_1 : \mu_{\text{Private}} \neq \mu_{\text{Public}}$. (We wish to determine whether the starting salaries of high-school math teachers in private schools are different from those of high-school math teachers in public schools.) What is the p-value for this test?

**5.** Six children are tested for pulse rate before and after watching a violent movie with the following results.

| Child | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Before | 102 | 96 | 89 | 104 | 90 | 85 |
| After | 109 | 105 | 94 | 112 | 104 | 96 |

Using the paired $t$ test, test for differences in the before and after mean pulse rates. Use $\alpha = 0.05$ and use a two-sided test.