STAT 400 Homework 09

Spring 2018 | Dalpiaz | UIUC

Due: *Friday, April 6, 2:00 PM*

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0, \ \theta > 0$$

Note that, the moments of this distribution are given by

$$E[X^k] = \int_0^\infty \frac{x^k}{\theta} e^{-x/\theta} = k! \cdot \theta^k.$$

This will be a useful fact for Exercises 2 and 3.

(a) Obtain the maximum likelihood *estimator* of θ, $\hat{\theta}$. (This should be a function of the unobserved x_i and the sample size n.) Calculate the *estimate* when

$$x_1 = 0.50, \ x_2 = 1.50, \ x_3 = 4.00, \ x_4 = 3.00.$$

(This should be a single number, for this dataset.)

(b) Calculate the bias of the maximum likelihood *estimator* of θ, $\hat{\theta}$. (This will be a number.)

(c) Find the mean squared error of the maximum likelihood *estimator* of θ, $\hat{\theta}$. (This will be an expression based on the parameter θ and the sample size n. Be aware of your answer to the previous part, as well as the distribution given.)

(d) Provide an *estimate* for $P[X > 4]$ when

$$x_1 = 0.50, \ x_2 = 1.50, \ x_3 = 4.00, \ x_4 = 3.00.$$

Exercise 2

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x, \alpha) = \alpha^{-2} x e^{-x/\alpha}, \quad x > 0, \ \alpha > 0$$

(a) Obtain the maximum likelihood *estimator* of α, $\hat{\alpha}$. Calculate the *estimate* when

$$x_1 = 0.25, \ x_2 = 0.75, \ x_3 = 1.50, \ x_4 = 2.5, \ x_5 = 2.0.$$

(b) Obtain the method of moments *estimator* of α, $\tilde{\alpha}$. Calculate the *estimate* when

$$x_1 = 0.25,\ x_2 = 0.75,\ x_3 = 1.50,\ x_4 = 2.5,\ x_5 = 2.0.$$

Exercise 3

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x, \beta) = \frac{1}{2\beta^3} x^2 e^{-x/\beta}, \quad x > 0,\ \beta > 0$$

(a) Obtain the maximum likelihood *estimator* of β, $\hat{\beta}$. Calculate the *estimate* when

$$x_1 = 2.00,\ x_2 = 4.00,\ x_3 = 7.50,\ x_4 = 3.00.$$

(b) Obtain the method of moments *estimator* of β, $\tilde{\beta}$. Calculate the *estimate* when

$$x_1 = 2.00,\ x_2 = 4.00,\ x_3 = 7.50,\ x_4 = 3.00.$$

Exercise 4

Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a distribution with probability density function

$$f(x, \lambda) = \lambda x^{\lambda - 1}, \quad 0 < x < 1, \lambda > 0$$

(a) Obtain the maximum likelihood *estimator* of λ, $\hat{\lambda}$. Calculate the *estimate* when

$$x_1 = 0.10,\ x_2 = 0.20,\ x_3 = 0.30,\ x_4 = 0.40.$$

(b) Obtain the method of moments *estimator* of λ, $\tilde{\lambda}$. Calculate the *estimate* when

$$x_1 = 0.10,\ x_2 = 0.20,\ x_3 = 0.30,\ x_4 = 0.40.$$