STAT 400 **Discussion 04 Solutions** Spring 2018

1. Suppose that the probability that a duck hunter will successfully hit a duck is 0.40 on any given shot. Suppose also that the outcome of each shot is independent from the others.

a) What is the probability that the first successful hit would be on the fourth shot?

Miss Miss Miss Hit

$0.60 \times 0.60 \times 0.60 \times 0.40 = \mathbf{0.0864}$.

Geometric distribution, $p = 0.40$.

$P(X = 4) = (1 - 0.40)^{4-1}\, 0.40 = \mathbf{0.0864}$.

b) What is the probability that it would take at least six shots to hit a duck for the first time?

For Geometric(p), $P(X > a) = (1 - p)^a, \quad a = 0, 1, 2, \ldots$.

$P(X \ge 6) = P(X > 5) = 0.60^5 = \mathbf{0.07776}$.

OR

$P(X \ge 6) = 1 - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4) - P(X = 5)$

$= 1 - 0.60^0\, 0.40 - 0.60^1\, 0.40 - 0.60^2\, 0.40 - 0.60^3\, 0.40 - 0.60^4\, 0.40$

$= 1 - 0.40 - 0.24 - 0.144 - 0.0864 - 0.05184$

$= 1 - 0.92224 = \mathbf{0.07776}$.

c) What is the probability that the first successful hit would happen during an even-numbered shot?

$$\text{P(even)} = \text{P}(2) + \text{P}(4) + \text{P}(6) + \ldots = 0.40 \cdot 0.60^{1} + 0.40 \cdot 0.60^{3} + 0.40 \cdot 0.60^{5} + \ldots$$

$$= \sum_{k=0}^{\infty} 0.24 \cdot 0.60^{2k} = 0.24 \cdot \sum_{n=0}^{\infty} 0.36^{n} = 0.24 \cdot \frac{1}{1-0.36} = \frac{24}{64} = \mathbf{\frac{3}{8}} = \mathbf{0.375}.$$

OR

$$\text{P(odd)} = 0.40 \cdot 0.60^{1} + 0.40 \cdot 0.60^{3} + 0.40 \cdot 0.60^{5} + \ldots$$

$$\text{P(even)} = 0.40 \cdot 0.60^{0} + 0.40 \cdot 0.60^{2} + 0.40 \cdot 0.60^{4} + \ldots$$

$\Rightarrow$ P(even) = 0.60 · P(odd). $\qquad$ $\text{P(odd)} = \frac{5}{3} \cdot \text{P(even)}.$

$\Rightarrow$ $1 = \text{P(odd)} + \text{P(even)} = \frac{8}{3} \cdot \text{P(even)}.$ $\qquad$ $\text{P(even)} = \mathbf{\frac{3}{8}} = \mathbf{0.375}.$

d) What is the probability that the third successful hit would be on the ninth shot?

Negative Binomial distribution, $p = 0.40$, $r = 3$.

$$\begin{array}{ll} [\text{ 8 shots: 2 S's \& 6 F's }] & \text{S} \\ \left[\binom{8}{2} \cdot (0.40)^{2} \cdot (0.60)^{6} \right] & \cdot\, 0.40 \approx \mathbf{0.0836}. \end{array}$$

OR

S S F F F F F F S	F S S F F F F F S	F F S F S F F F S	F F F S F F F S S
S F S F F F F F S	F S F S F F F F S	F F S F F S F F S	F F F F S S F F S
S F F S F F F F S	F S F F S F F F S	F F S F F F S F S	F F F F S F S F S
S F F F S F F F S	F S F F F S F F S	F F S F F F F S S	F F F F S F F S S
S F F F F S F F S	F S F F F F S F S	F F F S S F F F S	F F F F F S S F S
S F F F F F S F S	F S F F F F F S S	F F F S F S F F S	F F F F F S F S S
S F F F F F F S S	F F S S F F F F S	F F F S F F S F S	F F F F F F S S S

$28 \cdot (0.40)^{3} \cdot (0.60)^{6} \approx \mathbf{0.0836}.$

e) What is the probability that the hunter would have three successful hits in nine shots?

Let X = the number of successful hits in 9 shots.

Then X has Binomial distribution, $n = 9$, $p = 0.40$.

Need $\mathrm{P}(X = 3) = ?$ $\qquad\qquad \mathrm{P}(X = k) = \binom{n}{k} p^{\,k} (1-p)^{\,n-k}$.

$\mathrm{P}(X = 3) = \binom{9}{3} (0.40)^{3} (0.60)^{6} = \mathbf{0.2508}$.

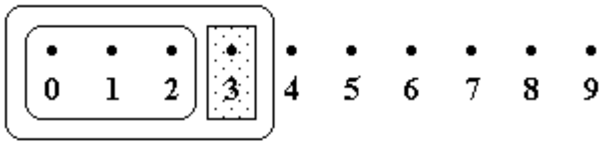
OR

S S F F F F F F S	F S S F F F F F S	F F S F S F F F S	F F F S F F F S S
S F S F F F F F S	F S F S F F F F S	F F S F F S F F S	F F F F S S F F S
S F F S F F F F S	F S F F S F F F S	F F S F F F S F S	F F F F S F S F S
S F F F S F F F S	F S F F F S F F S	F F S F F F F S S	F F F F S F F S S
S F F F F S F F S	F S F F F F S F S	F F F S S F F F S	F F F F F S S F S
S F F F F F S F S	F S F F F F F S S	F F F S F S F F S	F F F F F S F S S
S F F F F F F S S	F F S S F F F F S	F F F S F F S F S	F F F F F F S S S

S S S F F F F F F	S F F S F F F S F	F S F S F F S F F	F F S F S F F S F
S S F S F F F F F	S F F F S S F F F	F S F S F F F S F	F F S F F S S F F
S S F F S F F F F	S F F F S F S F F	F S F F S S F F F	F F S F F S F S F
S S F F F S F F F	S F F F S F F S F	F S F F S F S F F	F F S F F F S S F
S S F F F F S F F	S F F F F S S F F	F S F F S F F S F	F F F S S S F F F
S S F F F F F S F	S F F F F S F S F	F S F F F S S F F	F F F S S F S F F
S F S S F F F F F	S F F F F F S S F	F S F F F S F S F	F F F S S F F S F
S F S F S F F F F	F S S S F F F F F	F S F F F F S S F	F F F S F S S F F
S F S F F S F F F	F S S F S F F F F	F F S S S F F F F	F F F S F S F S F
S F S F F F S F F	F S S F F S F F F	F F S S F S F F F	F F F S F F S S F
S F S F F F F S F	F S S F F F S F F	F F S S F F S F F	F F F F S S S F F
S F F S S F F F F	F S S F F F F S F	F F S S F F F S F	F F F F S S F S F
S F F S F S F F F	F S F S S F F F F	F F S F S S F F F	F F F F S F S S F
S F F S F F S F F	F S F S F S F F F	F F S F S F S F F	F F F F F S S S F

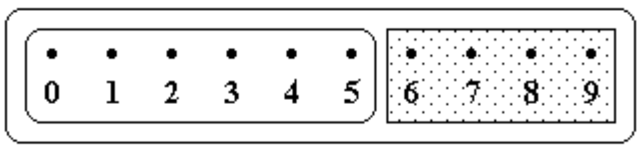
$84 \cdot (0.40)^{3} \cdot (0.60)^{6} \approx \mathbf{0.2508}$.

OR



$\mathrm{P}(X = 3) = \mathrm{P}(X \le 3) - \mathrm{P}(X \le 2)$ = CDF @ 3 – CDF @ 2 = 0.483 – 0.232 = **0.251**.

f) What is the probability that the hunter would have at least six successful hits in nine shots?



$\mathrm{P}(X \ge 6) = 1 - \mathrm{P}(X \le 5)$ = 1 – CDF @ 5 = 1 – 0.901 = **0.099**.

OR

$$\mathrm{P}(X \ge 6) = \binom{9}{6}(0.40)^6(0.60)^3 + \binom{9}{7}(0.40)^7(0.60)^2$$

$$+ \binom{9}{8}(0.40)^8(0.60)^1 + \binom{9}{9}(0.40)^9(0.60)^0$$

$$\approx 0.07432 + 0.02123 + 0.00354 + 0.00026 = \mathbf{0.09935}.$$

2. According to a *CNN/USA Today* poll, approximately 70% of Americans believe the IRS abuses its power. Let X denote the number of people who believe the IRS abuses its power in a random sample of $n = 50$ Americans. Assuming that the poll results are still valid, use a computer to find the probability that …

a) X is at most 32. b) X is equal to 34. c) X is at least 36.

Hint:			
	EXCEL:	=BINOMDIST$(k, n, p, 0)$	=BINOMDIST$(k, n, p, 1)$
	R:	> dbinom(k, n, p)	> pbinom(k, n, p)
	gives	$P(X = k)$	$P(X \le k)$

a) X is at most 32.

=BINOMDIST(32,50,0.7,1)

0.217807

```
> pbinom(32,50,0.7)
[1] 0.2178069
```

b) X is equal to 34.

=BINOMDIST(34,50,0.7,0)

0.1147

```
> dbinom(34,50,0.7)
[1] 0.1147002
```

c) X is at least 36.

=1-BINOMDIST(35,50,0.7,1)

0.446832

```
> 1-pbinom(35,50,0.7)
[1] 0.4468316
```

3. **2.4-11** **2.4-11**

A random variable X has a binomial distribution with mean 6 and variance 3.6. Find $P(X = 4)$.

$\mu = np = 6.$ $\sigma 2 = np(1-p) = 3.6.$

$\Rightarrow$ $1 - p = 0.60.$ $p = 0.40.$ $n = 15.$

$\Rightarrow$ $P(X = 4) = {}_{15}C_4 \cdot (0.40)^4 \cdot (0.60)^{11} \approx \mathbf{0.126776}.$

4. Alex sells "***Exciting World of Statistics***" videos over the phone to earn some extra cash during the economic crisis. Only 10% of all calls result in a sale. Assume that the outcome of each call is independent of the others.

a) What is the probability that Alex makes his first sale on the fifth call?

No sale		No sale		No sale		No sale		Sale		
(0.90)	•	(0.90)	•	(0.90)	•	(0.90)	•	(0.10)	=	**0.06561**.

Geometric distribution, $p = 0.10$.

b) What is the probability that Alex makes his first sale on an odd-numbered call?

$$\text{P(odd)} = \text{P}(1) + \text{P}(3) + \text{P}(5) + \ldots = 0.10 \cdot 0.90^{\,0} + 0.10 \cdot 0.90^{\,2} + 0.10 \cdot 0.90^{\,4} + \ldots$$

$$= \sum_{k=0}^{\infty} 0.10 \cdot 0.90^{\,2k} = 0.10 \cdot \sum_{n=0}^{\infty} 0.81^{\,n} = 0.10 \cdot \frac{1}{1-0.81} = \mathbf{\frac{10}{19}} \approx \mathbf{0.5263}.$$

OR

$$\text{P(odd)} = 0.10 \cdot 0.90^{\,0} + 0.10 \cdot 0.90^{\,2} + 0.10 \cdot 0.90^{\,4} + \ldots$$

$$\text{P(even)} = 0.10 \cdot 0.90^{\,1} + 0.10 \cdot 0.90^{\,3} + 0.10 \cdot 0.90^{\,5} + \ldots$$

$\Rightarrow$ P(even) = 0.90 · P(odd).

$\Rightarrow$ 1 = P(odd) + P(even) = 1.9 · P(odd). P(odd) = $\mathbf{\frac{10}{19}} \approx \mathbf{0.5263}$.

c) What is the probability that it takes Alex at least 10 calls to make his first sale?

For Geometric distribution,

X = number of independent attempts needed to get the first "success".

$\text{P}(\text{X} > a) = \text{P}(\text{the first } a \text{ attempts are "failures"}) = (1-p)^a, \quad a = 0, 1, 2, 3, \ldots$.

$\text{P}(\text{X} \geq 10) = \text{P}(\text{X} > 9) = 0.90^9 \approx \mathbf{0.38742}$.

d) What is the probability that it takes Alex at most 6 calls to make his first sale?

$$\mathrm{P}(\mathrm{X} \le 6) = 1 - \mathrm{P}(\mathrm{X} > 6) = 1 - 0.90^{6} = \mathbf{0.468559}.$$

e) What is the probability that Alex makes his second sale on the ninth call?

$$\underbrace{\left[\ {}_{8}C_{1} \cdot (0.10)^{1} \cdot (0.90)^{7}\ \right]}_{[\text{ 8 calls: 1 S \& 7 F's }]} \cdot \underbrace{0.10}_{\text{S}} \approx \mathbf{0.038264}.$$

OR

Let Y = the number of (independent) calls needed to make 2 sales.

$\Rightarrow$ Negative Binomial distribution, $k = 2$, $p = 0.10$.

$$\mathrm{P}(Y = y) = {}_{y-1}C_{k-1} \cdot p^{k} \cdot (1-p)^{y-k}$$

$$\mathrm{P}(Y = 13) = {}_{8}C_{1} \cdot (0.10)^{2} \cdot (0.90)^{7} \approx \mathbf{0.038264}.$$

f)* What is the probability that Alex makes his second sale on an odd-numbered call?

Hint: Consider $[\text{Answer}] - 0.9^{2} \times [\text{Answer}]$.
On one side, you will have $0.19 \times [\text{Answer}]$.
On the other side, you will have a geometric series.

Let Y = the number of (independent) calls needed to make 2 sales.

$\Rightarrow$ Negative Binomial distribution, $k = 2$, $p = 0.10$.

$$\mathrm{P}(Y = y) = {}_{y-1}C_{1} \cdot p^{k} \cdot (1-p)^{y-k} = (y-1) \cdot p^{k} \cdot (1-p)^{y-k}, \qquad y = 2, 3, 4, 5, \dots .$$

$$[\text{Answer}] = f(3) + f(5) + f(7) + f(9) + \dots$$
$$= 2 \cdot 0.10^{2} \cdot 0.90^{1} + 4 \cdot 0.10^{2} \cdot 0.90^{3} + 6 \cdot 0.10^{2} \cdot 0.90^{5} + 8 \cdot 0.10^{2} \cdot 0.90^{7} + \dots$$

$$0.81 \times [\text{Answer}] = 0.9^{2} \times [\text{Answer}]$$
$$= \qquad 2 \cdot 0.10^{2} \cdot 0.90^{3} + 4 \cdot 0.10^{2} \cdot 0.90^{5} + 6 \cdot 0.10^{2} \cdot 0.90^{7} + \dots$$

$$0.19 \times [\text{Answer}] = [\text{Answer}] - 0.81 \times [\text{Answer}]$$

$$= 2 \cdot 0.10^2 \cdot 0.90^1 + 2 \cdot 0.10^2 \cdot 0.90^3 + 2 \cdot 0.10^2 \cdot 0.90^5 + 2 \cdot 0.10^2 \cdot 0.90^7 + \ldots$$

$$= \frac{\textit{first term}}{1 - \textit{base}} = \frac{2 \cdot 0.10^2 \cdot 0.90^1}{1 - 0.90^2} = \frac{0.018}{0.19}.$$

$$[\text{Answer}] = \frac{0.018}{0.19^2} = \frac{0.018}{0.0361} = \mathbf{\frac{180}{361}} \approx \mathbf{0.498615}.$$

g) What is the probability that Alex makes his third sale on the 13th call?

$$[\text{ 12 calls: 2 S's \& 10 F's }] \qquad \text{S}$$

$$\left\lfloor {}_{12}C_2 \cdot (0.10)^2 \cdot (0.90)^{10} \right\rfloor \cdot 0.10 = \mathbf{0.0230}.$$

OR

Let Y = the number of (independent) calls needed to make 3 sales.

$\Rightarrow$ Negative Binomial distribution, $k = 3$, $p = 0.10$.

$$\text{P}(Y = y) = {}_{y-1}C_{k-1} \cdot p^k \cdot (1-p)^{y-k}$$

$$\text{P}(Y = 13) = {}_{12}C_2 \cdot (0.10)^3 \cdot (0.90)^{10} = \mathbf{0.0230}.$$

h) If Alex makes 15 calls, what is the probability that he makes exactly 3 sales?

Let X = the number of sales made during 15 phone calls.

The outcome of each call is independent of the others

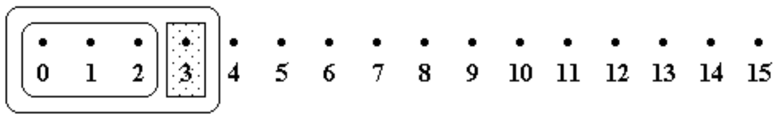
$\Rightarrow$ Binomial distribution, $n = 15$, $p = 0.10$.

Need $\text{P}(X = 3) = ?$ $\qquad \text{P}(X = k) = {}_nC_k \cdot p^k \cdot (1-p)^{n-k}$

$$\text{P}(X = 3) = {}_{15}C_3 \cdot (0.10)^3 \cdot (0.90)^{12} = \mathbf{0.1285}.$$

OR

Using Cumulative Binomial Probabilities:



P(X = 3) = P($X \le 3$) – P($X \le 2$) = CDF @ 3 – CDF @ 2 = 0.944 – 0.816 = **0.128**.

i) If Alex makes 15 calls, what is the probability that he makes at least 2 sales?

Need P($X \ge 2$) = ?

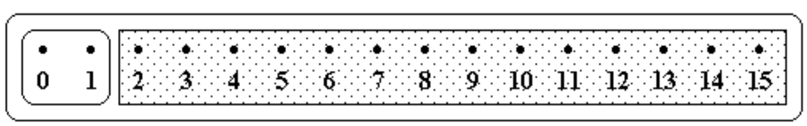
P($X \ge 2$) = 1 – P(X = 0) – P(X = 1)

$$= 1 - {}_{15}C_0 \cdot (0.10)^0 \cdot (0.90)^{15} - {}_{15}C_1 \cdot (0.10)^1 \cdot (0.90)^{14}$$

$$= 1 - 0.2059 - 0.3432 = \mathbf{0.4509}.$$

OR

Using Cumulative Binomial Probabilities:



P($X \ge 2$) = 1 – P($X \le 1$) = 1 – CDF @ 1 = 1 – 0.549 = **0.451**.

j) If Alex makes 15 calls, what is the probability that he makes at most 2 sales?

$$P(X \le 2) = {}_{15}C_0 \cdot (0.10)^0 \cdot (0.90)^{15} + {}_{15}C_1 \cdot (0.10)^1 \cdot (0.90)^{14} + {}_{15}C_2 \cdot (0.10)^2 \cdot (0.90)^{13}$$

$$= 0.2059 + 0.3432 + 0.2669 = \mathbf{0.8160}.$$

OR

P($X \le 2$) = CDF @ 2 = **0.816**.

5. A grocery store has 10 loaves of bread on its shelves, of which 7 are fresh and 3 are stale. Customers buy 4 loaves selecting them at random.

a) Find the probability that 3 are fresh and 1 is stale.

$$\frac{{}_7C_3 \cdot {}_3C_1}{{}_{10}C_4} = \mathbf{0.50}.$$

b) Find the probability that 2 are fresh and 2 are stale.

$$\frac{{}_7C_2 \cdot {}_3C_2}{{}_{10}C_4} = \mathbf{0.30}.$$

c) Find the probability that at least 2 loaves are fresh.

$$\frac{{}_7C_2 \cdot {}_3C_2}{{}_{10}C_4} + \frac{{}_7C_3 \cdot {}_3C_1}{{}_{10}C_4} + \frac{{}_7C_4 \cdot {}_3C_0}{{}_{10}C_4} \approx \mathbf{0.96667}.$$

6. According to news reports in early 1995, among the first Pentium chips Intel made, some had a peculiar defect, which rendered some rarely carried-out arithmetic operations incorrect. Any chip could therefore be classified into one of three categories: Good, Broken (useless), or Defective (operable except for the peculiar defect described above). Suppose that 70% of the chips made were good, 25% had a peculiar defect, and 5% were broken. If a random sample of 20 chips was selected, what is the probability that 15 were good, 3 defective, and 2 broken?

$$\frac{20!}{15! \cdot 3! \cdot 2!} \cdot 0.70^{15} \cdot 0.25^{3} \cdot 0.05^{2} \approx \mathbf{0.0287524}.$$

7. a) **2.6-2** **2.6-2**

Let X have a Poisson distribution with variance of 3. Find $P(X = 2)$.

Poisson distribution: $\operatorname{Var}(X) = \lambda \quad \Rightarrow \quad \lambda = 3$

Thus, $P(X = 2) = \dfrac{\lambda^2 e^{-\lambda}}{2!} = \dfrac{3^2 e^{-3}}{2!} = \mathbf{0.22404}$.

OR

Table III $\quad P(X = 2) = P(X \le 2) - P(X \le 1) = 0.423 - 0.199 = \mathbf{0.224}$.

b) **2.6-4** **2.6-4**

If X has a Poisson distribution such that $3\,P(X = 1) = P(X = 2)$, find $P(X = 4)$.

$$3 \cdot \frac{\lambda^1 e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!} \quad \Rightarrow \quad 6\lambda = \lambda^2$$

$$\Rightarrow \quad \lambda = 6 \qquad \text{since } \lambda > 0.$$

Thus, $P(X = 4) = \dfrac{\lambda^4 e^{-\lambda}}{4!} = \dfrac{6^4 e^{-6}}{4!} = \mathbf{0.13385}$.

OR

Table III $\quad P(X = 4) = P(X \le 4) - P(X \le 3) = 0.285 - 0.151 = \mathbf{0.134}$.

8. Suppose the number of air bubbles in window glass has Poisson distribution, with an average of 0.3 air bubbles per square foot. Find the probability of finding in a 4' by 3' window …

4' by 3' = 12 square feet. $\lambda = 0.3 \times 12 = 3.6$.

a) … exactly 5 air bubbles.

$$P(X=5) = \frac{3.6^5 \cdot e^{-3.6}}{5!} = \mathbf{0.13768}.$$

OR

Table III $P(X=5) = P(X \le 5) - P(X \le 4) = 0.844 - 0.706 = \mathbf{0.138}$.

b) … at least 5 air bubbles.

Table III $P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.706 = \mathbf{0.294}$.

OR

$$\frac{3.6^0 \cdot e^{-3.6}}{0!} + \frac{3.6^1 \cdot e^{-3.6}}{1!} + \frac{3.6^2 \cdot e^{-3.6}}{2!} + \frac{3.6^3 \cdot e^{-3.6}}{3!} + \frac{3.6^4 \cdot e^{-3.6}}{4!}$$

$$= 0.02732 + 0.09837 + 0.17706 + 0.21247 + 0.19122 = 0.70644.$$

$P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.70644 = \mathbf{0.29356}$.

9. **2.6-8** **2.6-8**

Suppose that the probability of suffering a side effect from a certain flu vaccine is 0.005. If 1000 persons are inoculated, find the approximate probability that

a) At most 1 person suffers.

(use Poisson approximation)

b) 4, 5, or 6 persons suffer.

2.6-8 $n\,p = 1000\,(0.005) = 5.$

(a) $P(X \le 1) = P(X = 0) + P(X = 1).$

Binomial

$$P(X=0) = \binom{1000}{0}(0.005)^0(0.995)^{1000} = 0.006654;$$

$$P(X=1) = \binom{1000}{1}(0.005)^1(0.995)^{999} = 0.033437;$$

$$0.006654 + 0.033437 = 0.040091.$$

Poisson

$$P(X=0) = \frac{5^0 \cdot e^{-5}}{0!} = 0.006738;$$

$$P(X=1) = \frac{5^1 \cdot e^{-5}}{1!} = 0.033690;$$

$$0.006738 + 0.033690 = 0.040428.$$

(b) $P(X = 4, 5, 6) = P(X = 4) + P(X = 5) + P(X = 6).$

Binomial

$$P(X=4) = \binom{1000}{4}(0.005)^4(0.995)^{996} = 0.17573;$$

$$P(X=5) = \binom{1000}{5}(0.005)^5(0.995)^{995} = 0.17591;$$

$$P(X=6) = \binom{1000}{6}(0.005)^6(0.995)^{994} = 0.14659;$$

$$0.17573 + 0.17591 + 0.14659 = 0.49823.$$

Poisson

$$P(X=4) = \frac{5^4 \cdot e^{-5}}{4!} = 0.17547;$$

$$P(X=5) = \frac{5^5 \cdot e^{-5}}{5!} = 0.17547;$$

$$P(X=6) = \frac{5^6 \cdot e^{-5}}{6!} = 0.14622;$$

$$0.17547 + 0.17547 + 0.14622 = 0.49716.$$

10. Urbana-Champaign Academics ($\cup\subset\forall$) is a semi-professional hockey team. Suppose that score goals according to a Poisson process with the average rate of 1 goal per 8 minutes. A hockey game consists of 3 periods, each lasting 20 minutes.

a) Find the probability that $\cup\subset\forall$ would score exactly 6 goals in one game.

one game = 60 minutes, average rate of 1 goal per 8 minutes, $\lambda = 7.5$.

exactly 6 goals in one game $\quad \dfrac{7.5^{6}\, e^{-7.5}}{6!} \approx \mathbf{0.1367}$.

b) Find the probability that $\cup\subset\forall$ would score exactly 2 goals in each period in one game.

one period = 20 minutes, average rate of 1 goal per 8 minutes, $\lambda = 2.5$.

exactly 2 goals in one period $\quad \dfrac{2.5^{2}\, e^{-2.5}}{2!} \approx 0.2565$.

exactly 2 goals in each period $\quad \left(\dfrac{2.5^{2}\, e^{-2.5}}{2!}\right)^{3} \approx \mathbf{0.0169}$.

c) Find the probability that $\cup\subset\forall$ would score exactly 2 goals in exactly 2 periods in one game.

Binomial distribution, $\quad n = 3, \quad p = \dfrac{2.5^{2}\, e^{-2.5}}{2!} \approx 0.2565$.

$$\binom{3}{2} \times \left(\frac{2.5^{2}\, e^{-2.5}}{2!}\right)^{2} \times \left(1 - \frac{2.5^{2}\, e^{-2.5}}{2!}\right)^{1} \approx \mathbf{0.1468}.$$

d) Find the probability that $\cup\subset\forall$ would score at least 1 goal in each period in one game.

one period = 20 minutes, average rate of 1 goal per 8 minutes, $\lambda = 2.5$.

no goals in one period $\dfrac{2.5^0 \, e^{-2.5}}{0!} \approx 0.0821.$

at least 1 goal in one period $1 - \dfrac{2.5^0 \, e^{-2.5}}{0!} \approx 0.9179.$

at least 1 goal in each period $\left(1 - \dfrac{2.5^0 \, e^{-2.5}}{0!}\right)^3 \approx \mathbf{0.7734}.$

e) Find the probability that $\cup\subset\forall$ would fail to score a goal in exactly 1 period in one game.

Binomial distribution, $n = 3, \quad p = \dfrac{2.5^0 \, e^{-2.5}}{0!} \approx 0.0821.$

$$\binom{3}{1} \times \left(\frac{2.5^0 \, e^{-2.5}}{0!}\right)^1 \times \left(1 - \frac{2.5^0 \, e^{-2.5}}{0!}\right)^2 \approx \mathbf{0.2075}.$$

f) Find the probability that $\cup\subset\forall$ would score at most 4 goals in one game.

one game = 60 minutes, average rate of 1 goal per 8 minutes, $\lambda = 7.5$.

Table III $\mathrm{P}(X \le 4) = \mathbf{0.132}.$

$$\frac{7.5^0 \, e^{-7.5}}{0!} + \frac{7.5^1 \, e^{-7.5}}{1!} + \frac{7.5^2 \, e^{-7.5}}{2!} + \frac{7.5^3 \, e^{-7.5}}{3!} + \frac{7.5^4 \, e^{-7.5}}{4!} \approx \mathbf{0.1321}.$$

11. The number of typos made by a student follows Poisson distribution with the rate of 1.5 typos per page. Assume that the numbers of typos on different pages are independent.

a) Find the probability that there are at most 2 typos on a page.

1 page $\Rightarrow$ $\lambda = 1.5$.

Need $P(X \le 2) = ?$ Poisson distribution: $P(X = x) = \dfrac{\lambda^x \cdot e^{-\lambda}}{x!}$

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \frac{1.5^0 \cdot e^{-1.5}}{0!} + \frac{1.5^1 \cdot e^{-1.5}}{1!} + \frac{1.5^2 \cdot e^{-1.5}}{2!}$$

$$= 0.2231 + 0.3347 + 0.2510 = \mathbf{0.8088}.$$

b) Find the probability that there are exactly 10 typos in a 5-page paper.

5 pages $\Rightarrow$ $\lambda = 1.5 \cdot 5 = 7.5$. $P(X = 10) = \dfrac{7.5^{10} \cdot e^{-7.5}}{10!} = \mathbf{0.0858}.$

c) Find the probability that there are exactly 2 typos on each page in a 5-page paper.

Let $A_i = \{ 2 \text{ typos on } i^{\text{th}} \text{ page} \}$, $i = 1, 2, 3, 4, 5.$

Then $P(A_1) = P(A_2) = P(A_3) = P(A_4) = P(A_5) = 0.2510$ (Poisson, $\lambda = 1.5$)

$P(\text{2 typos on each page in a 5-page paper}) = P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5)$

by independence

$$= P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot P(A_4) \cdot P(A_5) = (0.2510)^5 \approx \mathbf{0.000996}.$$

OR

Let Y = the number of pages (out of 5) with exactly 2 typos.

Then Y has Binomial distribution, $n = 5$, $p = 0.2510$ (Poisson, λ = 1.5)

$$\text{P}(\text{ Y} = 5\,) = {}_5C_5 \cdot (0.2510)^5 \cdot (1 - 0.2510)^0 \approx \mathbf{0.000996}.$$

d) Find the probability that there is at least one page with no typos in a 5-page paper.

Let B_i = { no typos on i^{th} page }, i = 1, 2, 3, 4, 5.

Then $\text{P}(\text{ B}_1\,) = \text{P}(\text{ B}_2\,) = \text{P}(\text{ B}_3\,) = \text{P}(\text{ B}_4\,) = \text{P}(\text{ B}_5\,) = 0.2231$ (Poisson, λ = 1.5)

P(at least one page with no typos in a 5-page paper) = $\text{P}(\text{ B}_1 \cup \text{B}_2 \cup \text{B}_3 \cup \text{B}_4 \cup \text{B}_5\,)$

$$= 1 - \text{P}(\text{ B}_1' \cap \text{B}_2' \cap \text{B}_3' \cap \text{B}_4' \cap \text{B}_5'\,)$$

$$= 1 - \text{P}(\text{ B}_1'\,) \cdot \text{P}(\text{ B}_2'\,) \cdot \text{P}(\text{ B}_3'\,) \cdot \text{P}(\text{ B}_4'\,) \cdot \text{P}(\text{ B}_5'\,) = 1 - (0.7769)^5 \approx \mathbf{0.7170}.$$

OR

Let W = the number of pages (out of 5) with no typos.

Then W has Binomial distribution, $n = 5$, $p = 0.2231$ (Poisson, λ = 1.5)

$$\text{P}(\text{ W} \geq 1\,) = 1 - \text{P}(\text{ W} = 0\,) = 1 - {}_5C_0 \cdot (0.2231)^0 \cdot (1 - 0.2231)^5 \approx \mathbf{0.7170}.$$

e) Find the probability that there are exactly two pages with no typos in a 5-page paper.

Let W = the number of pages (out of 5) with no typos.

Then W has Binomial distribution, $n = 5$, $p = 0.2231$ (Poisson, λ = 1.5)

$$\text{P}(\text{ W} = 2\,) = {}_5C_2 \cdot (0.2231)^2 \cdot (1 - 0.2231)^3 = \mathbf{0.2334}.$$