STAT 400 | Spring 2018

Discussion 11

1. A random sample of size $n = 9$ from a normal $N(\mu, \sigma^2)$ distribution is obtained:

4.4 3.7 5.1 4.3 4.7 3.7 3.5 4.6 4.7

a) Compute the sample mean $\bar{x}$ and the sample standard deviation s.

b) Construct a 95% (two-sided) confidence interval for the overall (population) mean.

c) Construct a 90% one-sided confidence interval for μ that provides an upper bound for μ.

d) Construct a 95% one-sided confidence interval for μ that provides a lower bound for μ.

e) Construct a 95% (two-sided) confidence interval for the overall standard deviation.

f) Construct a 90% one-sided confidence interval for σ that provides an upper bound for σ.

g) Construct a 95% one-sided confidence interval for σ that provides a lower bound for σ.

2. An examination of the records for a random sample of 16 motor vehicles in a large fleet resulted in the sample mean operating cost of 26.33 cents per mile and the sample standard deviation of 2.80 cents per mile. (Assume that operating costs are approximately normally distributed.)

a) Construct a 95% confidence interval for the mean operating cost.

b) Construct a 90% confidence interval for the variance of the operating costs.

3. Suppose the time spent on a particular STAT 400 homework follows a normal distribution with an overall standard deviation of 28 minutes and an unknown mean.

a) Suppose a random sample of 49 students is obtained. Find the probability that the average time spent on the homework for students in the sample is within 5 minutes of the overall mean.

b) A sample of 49 students has a sample mean of 234 minutes spent on the homework. Construct a 90% confidence interval for the overall mean time spent on the homework.

c) What is the minimum sample size required if we want to estimate the overall mean time spent on the homework to within 5 minutes with 90% confidence?

4. An economist states that 10% of Springfield's labor force is unemployed. A random sample of 400 people in the labor force is obtained, of whom 28 are unemployed.

a) Construct a 95% confidence interval for the unemployment rate in Springfield.

b) What is the minimum sample size required in order to estimate the unemployment rate in Springfield to within 2% with 95% confidence? (Use the economist's guess.)

c) What is the minimum sample size required in order to estimate the unemployment rate in Springfield to within 2% with 95% confidence? (Assume no information is available.)

5. The proportion of defective items is not supposed to be over 15%. A buyer wants to test whether the proportion of defectives exceeds the allowable limit. The buyer takes a random sample of 100 items and finds that 19 are defective.

a) Construct a 95% confidence interval for the overall proportion of defective items.

b) What is the minimum sample size required in order to estimate the overall proportion of defective items to within 3% with 95% confidence? (Assume that the overall proportion of defective items is at most 0.20.)

6. A coffee machine is regulated so that the amount of coffee dispensed is normally distributed. A random sample of 17 cups is given below:

8.15	7.93	8.04	7.80	8.02	7.92
8.18	7.65	7.73	8.15	7.68	7.85
7.97	7.70	7.75	7.87	8.08	

a) Compute the sample mean and the sample standard deviation.

"Hint": EXCEL =AVERAGE(…) =SUM(…)
=STDEV(…) =VAR(…)

OR R

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> x = c( … )
> mean(x)        > sum(x)
> sd(x)          > var(x)
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b) Construct a 90% confidence interval for the overall average amount of coffee dispensed by the machine.

7. **7.1-10** **6.2-12**

A leakage test was conducted to determine the effectiveness of a seal designed To keep the inside of a plug airtight. An air needle was inserted into the plug, and the plug and needle were placed under water. The pressure was then increased until leakage was observed. Let X equal the pressure in pounds per square inch. Assume that X follows a normal distribution. The following 10 observations of X were recorded:

3.1 3.3 4.5 2.8 3.5 3.5 3.7 4.2 3.9 3.3

a) Find a point estimate of μ using the observations.

b) Find a point estimate of σ using the observations.

c) Find a 95% confidence upper bound for μ.

8. 7. continued

d) Construct a 95% (two-sided) confidence interval for σ.

e) Find a 95% confidence upper bound for σ.

9. Analysis of the venom of seven 8-day-old worker bees yielded the sample mean histamine content (nanograms) of 640, with sample standard deviation of 200. Construct a 90% confidence interval for average histamine content for all worker bees of this age. (Assume that the histamine content is approximately normally distributed.)

10. Suppose the IQs of students at Anytown State University are normally distributed with standard deviation 15 and unknown mean.

a) Suppose a random sample of 64 students is obtained. Find the probability that the average IQ of the students in the sample will be within 3 points of the overall mean.

b) A sample of 64 students had a sample mean IQ of 115. Construct a 95% confidence interval for the overall mean IQ of students at Anytown State University.

c) What is the minimum sample size required if we want to estimate the overall mean IQ of students at Anytown State University to within 3 points with 95% confidence?

d) Suppose that only 20% of the students at Anytown State University have the IQ above 130. Find the overall average IQ of the students.

"Hint": From now on, you have μ.

e) Find the probability that the sample average IQ will be 115 or higher for a random sample of 64 students.

f) Only students in the top 33% are allowed to join the science club. What is the minimum IQ required to be able to join the science club?

g) What proportion of the students have IQ of 127 or above?

h) Find the probability that exactly 13 out of 64 randomly and independently selected students have IQ of 127 or above.

11. In a highly publicized study, doctors claimed that aspirin seems to help reduce heart attacks rate. Suppose a group of 400 men from a particular age group took an aspirin tablet three times per week. After three years, 56 of them had had heart attacks. Let p denote the overall proportion of men (in this age group) who take aspirin that have heart attacks in a 3-year period.

a) Construct a 90% confidence interval for p.

b) Construct a 95% confidence interval for p.

c) Find the 99% confidence upper bound for p.

12.* (0) Let X have a $\chi^2(r)$ distribution. If $k > -r/2$, prove (show) that $E(X^k)$ exists and it is given by

$$E(X^k) = \frac{2^k \Gamma\left(\frac{r}{2}+k\right)}{\Gamma\left(\frac{r}{2}\right)}.$$

6.4-14 (a),(b) 6.1-14 (a),(b)

Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a $N(\mu, \sigma^2)$ distribution.

(a) Show that an unbiased estimator of σ is $c\,S$, where

$$c = \frac{\sqrt{n-1}\,\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{2}\,\Gamma\left(\frac{n}{2}\right)}.$$

Hint: Recall that $X = (n-1)S^2/\sigma^2$ has a $\chi^2(n-1)$ distribution.

“Hint”: Select the appropriate value for k in part (0).

(b) Find the value of c when $n = 5$; when $n = 6$.