STAT 400 Spring 2018

Discussion 12

1. A random sample of size $n = 9$ from a normal $N(\mu, \sigma^2)$ distribution is obtained:

4.4 3.7 5.1 4.3 4.7 3.7 3.5 4.6 4.7

Recall: $\bar{x} = 4.3$, $s = 0.55$.

h) Test $H_0: \mu = 4$ vs. $H_1: \mu > 4$ at a 5% level of significance.
What is the p-value of the test? (You may give a range.)

i) Test $H_0: \sigma = 0.4$ vs. $H_1: \sigma > 0.4$ at a 5% level of significance.
What is the p-value of the test? (You may give a range.)

2. An examination of the records for a random sample of 16 motor vehicles in a large fleet resulted in the sample mean operating cost of 26.33 cents per mile and the sample standard deviation of 2.80 cents per mile. (Assume that operating costs are approximately normally distributed.)

c) The manager wants to believe that the actual mean operating cost is at most 25 cents per mile. Perform the appropriate test at a 5% level of significance.

d) Using the *t* distribution table only, what is the p-value of the test in part (c)? (You may give a range.)

e) Use a computer to find the p-value of the test in part (c).

f) Test whether the overall standard deviation of the operating costs is more than 2.30 cents per mile or not at a 5% significance level.

g) Using the Chi-Square distribution table only, what is the p-value of the test in part (f)? (You may give a range.)

h) Use a computer to find the p-value of the test in part (f).

3. In a random sample of 160 students from Anytown State University, 56 believe that resume inflation is unethical.

a) Construct a 90% confidence interval for p, the overall proportion of Anytown State University students who believe that resume inflation is unethical.

b) Find the p-value of the test $\mathrm{H}_0 : p = 0.40$ vs. $\mathrm{H}_1 : p < 0.40$.

4. An economist states that 10% of Springfield's labor force is unemployed. A random sample of 400 people in the labor force is obtained, of whom 28 are unemployed.

d) Test whether Springfield's unemployment rate is as the economist claims or is different at a 1% level of significance

e) Find the p-value of the test in part (d).

f) Using the p-value from part (e), state your decision (Reject H_0 or Do Not Reject H_0) at $\alpha = 0.06$ and at $\alpha = 0.03$.

5. The proportion of defective items is not supposed to be over 15%. A buyer wants to test whether the proportion of defectives exceeds the allowable limit. The buyer takes a random sample of 100 items and finds that 19 are defective.

c) Perform the appropriate test at $\alpha = 0.05$.

d) Find the p-value of the test in part (c).

e) Using the p-value from part (d), state your decision (Reject H_0 or Do Not Reject H_0) at $\alpha = 0.10$.

6. A coffee machine is regulated so that the amount of coffee dispensed is normally distributed. A random sample of 17 cups is given below:

8.15	7.93	8.04	7.80	8.02	7.92
8.18	7.65	7.73	8.15	7.68	7.85
7.97	7.70	7.75	7.87	8.08	

Recall: $\bar{x} = 7.91,\quad s = 0.175.$

c) The coffee machine manufacturer claims that the overall average amount of coffee dispensed is at least 8 ounces. Use $\alpha = 0.05$ to perform the appropriate test. State the null and alternative hypothesis for this test in terms of the relevant parameter, report the value of the test statistic, the critical value(s), and state your decision.

d) Find the p-value (approximately) of the test in part (c).

7. A random sample of 144 patients, suffering from a particular disease are given a new medicine. 108 of the patients report an improvement in their condition.

g) The company that manufactures this medicine claims 80% improvement rate. We suspect that it is less than 80%. Use $\alpha = 0.05$ to perform the appropriate test. State the null and alternative hypothesis for this test in terms of the relevant parameter, report the value of the test statistic, the critical value(s), and state your decision.

h) Find the p-value (approximately) of the test in part (g).

8. Suppose that 36% of the high schools students in a particular school district tried smoking last year. This year, a series of negative commercials about teen smoking was shown on all local TV channels. While the majority of the local community thinks the commercials helped reduce the proportion of high school students who smoke, some members of the community claim that the commercials just peak students' curiosity about smoking and thus increase the proportion of students who smoke. In a random sample of 225 high school students, 72 tried smoking this year.

a) Construct a 95% confidence interval for p, the overall proportion of students who tried smoking this year.

b) We wish to test $H_0: p = 0.36$ vs. $H_1: p \neq 0.36$. Find the p-value for this test.

9. Several employees of *Al's Building Construction* complain that the variance of the employees' monthly salary amounts is too large. In a random sample of 10 employees, the sample standard deviation of the monthly salary amounts was $210.

a) Use $\alpha = 0.05$ to test $H_0 : \sigma \le 150$ vs. $H_1 : \sigma > 150$. Report the value of the test statistic, the critical value(s), and state your decision.

b) Using the chi-square distribution table only, what is the p-value of the test in part (a)? (You may give a range.)

c) Use a computer to find the p-value of the test in part (a).

10. A contractor assumes that construction workers are idle for at most 75 minutes per day (on average). A sample of 16 construction workers had a mean of 83 minutes per day. The sample standard deviation was 20 minutes. Assume that the time the workers are idle is approximately normally distributed.

a) Use $\alpha = 0.05$ to perform the appropriate test.

b) What is the p-value of the test in part (a)? (You may give a range.)

c) Use a computer to find the p-value of the test in part (a).

11. The service time in queues should not have a large variance; otherwise, the queue tends to build up. A bank regularly checks the service times of its tellers to determine their variance. A random sample of 22 service times (in minutes) gives $s^2 = 8$. Assume the service times are normally distributed.

a) Find a 95% confidence interval for the overall variance of service time at the bank.

b) Find the two one-sided 95% confidence intervals for the overall variance of service time at the bank.

c) Test $H_0 : \sigma^2 = 5$ vs. $H_1 : \sigma^2 > 5$ at a 5% level of significance.

d) Using the Chi-Square distribution table only, what is the p-value of the test in part (c)? (You may give a range.)

e) Use a computer to find the p-value of the test in part (c).

12. The label on 1-gallon can of paint states that the amount of paint in the can is sufficient to paint at least 350 square feet (on average). Suppose the amount of coverage is approximately normally distributed, and the overall standard deviation of the amount of coverage is 32 square feet. A random sample of 16 cans yields the sample mean amount of coverage of 338 square feet. We wish to test

$$H_0 : \mu = 350 \quad \text{against} \quad H_1 : \mu < 350.$$

a) Find the p-value for the test.

b) Find the Rejection Region for the test at $\alpha = 0.05$. That is, for which values of the sample mean $\overline{X}$ should we reject H_0, if a 5% level of significance is used?

c) State your decision, Reject H_0 or Do NOT Reject H_0, at $\alpha = 0.05$.

13. In a highly publicized study, doctors claimed that aspirin seems to help reduce heart attacks rate. Suppose a group of 400 men from a particular age group took an aspirin tablet three times per week. After three years, 56 of them had had heart attacks. Let p denote the overall proportion of men (in this age group) who take aspirin that have heart attacks in a 3-year period.

d) Find the p-value of the test $H_0 : p \ge 0.17$ vs. $H_1 : p < 0.17$.

e) Using the p-value from part (d), state your decision (Reject H_0 or Do Not Reject H_0) for $\alpha = 0.05$.

f) Suppose that the true proportion of all aspirin-taking men in this age group who would have a heart attack over the next 3 years is 0.15. Then in part (e) [Type I Error, Type II Error, correct decision] was made. (*Choose one and justify your answer.*)

14. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x;\theta) = \frac{12}{\theta^4} x^2 (\theta - x), \qquad 0 < x < \theta, \qquad \theta > 0.$$

Find a method of moments estimator of θ, $\tilde{\theta}$.

15. Let $\gamma > 1$ and let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the distribution with probability density function

$$f(x;\gamma) = \frac{(\gamma-1)^4 (\ln x)^3}{6x^\gamma}, \qquad x > 1, \qquad \text{zero otherwise.}$$

Find the maximum likelihood estimator $\hat{\gamma}$ of γ.

Suppose $n = 3$, and $x_1 = 2$, $x_2 = 2.5$, $x_3 = 4$.

Find the maximum likelihood estimate of γ.

16. Let $\lambda > 0$. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x;\lambda) = -\lambda^2 \ln x \cdot x^{\lambda-1}, \qquad 0 < x < 1, \qquad \text{zero otherwise.}$$

Note: Since $0 < x < 1$, $\ln x < 0$.

a) Obtain the maximum likelihood estimator of λ, $\hat{\lambda}$.

b) Obtain a method of moments estimator of λ, $\tilde{\lambda}$.

17. Let X and Y have the joint probability density function

$$f(x,y) = \frac{Cy}{x^4}, \qquad y > 2, \quad x > 3y, \qquad \text{zero otherwise.}$$

a) Sketch the support of (X, Y). That is, sketch $\{ y > 2,\ x > 3y \}$.

b) Find the value of C that would make $f(x,y)$ is a valid joint p.d.f.

c) Set up the integral(s) for $P(X + Y > 16)$.

d) Set up the integral(s) for $P(X \div Y < 5)$.

e) Find the marginal probability density function of X, $f_X(x)$. *Include its support.*

f) Find the marginal probability density function of Y, $f_Y(y)$. *Include its support.*