STAT 400 AL1
Spring 2018
Exam I
3/1/2018
Time Limit: 75 Minutes
Version: A B C

Last Name: KEY

First Name:

NetID

Discussion CIRCLE BELOW

AD1	**AD2**	**AD3**	**AD4**
Wed 2:00	Wed 4:00	Thu 11:00	Thu 2:00
Neha	Josh	Neha	Josh

This exam contains **12 pages** which include: this cover page, exam material, one page of scratch paper, and a standard normal table. There are a total of **9 problems**. Check to see if any problems are missing. You may detach the tables. Do not turn in your tables if you have detached them. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Write your final answer in the space provided** using **four** significant digits where applicable.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you engage in an act of academic dishonesty, you become liable to severe disciplinary action. Such acts include cheating; falsification or invention of information or citation in an academic endeavor; helping or attempting to help others commit academic infractions; plagiarism; offering bribes, favors, or threats; academic interference; computer related infractions; and failure to comply with research regulations.
- Do not mark on the exam after time has been called.

Problem	Points	Score
1	10	
2	12	
3	12	
4	13	
5	14	
6	14	
7	6	
8	5	
9	14	
Total:	100	

A

1. (10 points) For each part, calculate the requested probability.

(a) (2 points) Let $A \sim \text{binom}(n = 10, p = 0.2)$. Calculate $P[A = 2]$.

$$P[A=2] = \binom{10}{2}(0.2)^2(0.8)^8 =$$

(a) 0.3020

(b) (2 points) Let $B \sim \text{pois}(\lambda = 7)$. Calculate $P[B = 5]$.

$$P[B=5] = \frac{7^5 e^{-7}}{5!} =$$

(b) 0.1277

(c) (2 points) Let $C \sim \text{geom}(p = 0.1)$. Calculate $P[C = 4]$.

$$P[C=4] = (0.9)^3(0.1) =$$

(c) 0.0729

(d) (2 points) Let $D \sim \text{N}(\mu = 2, \sigma^2 = 1)$. Calculate $P[D < 3]$.

$$P[D<3] = P[Z<1] =$$

(d) 0.8413

(e) (2 points) Let $E \sim \text{N}(\mu = 0, \sigma^2 = 1)$. Calculate $P[E = 1]$.

$$P[E=1] = 0 =$$

(e) 0

A

2. (12 points) Suppose the moment-generating function of a random variable X is

$$M_X(t) = (0.6e^t + 0.4)^4. \implies X \sim b(n=4, p=0.6)$$

(a) (4 points) Calculate the expected value of X, E[X].

$E[x] = np = 4(0.6)$

OR CALCULATE $M'_x(0)$

(a) 2.4

(b) (4 points) Calculate the second moment of X, E[X^2].

$VAR[x] = E[x^2] - (E[x])^2$

$E[x^2] = VAR[x] + (E[x])^2 = 0.96 + (2.4)^2 =$

OR CALCULATE $M''_x(0)$

(b) 6.72

(c) (4 points) Calculate the standard deviation of X, SD[X].

$VAR[x] = np(1-p) = 4(0.6)(0.4) = 0.96$

$SD[x] = \sqrt{0.96} =$

(c) 0.9798

B

3. (6 points) You are a Pokémon trainer that encounters a wild Snorlax. The encounter is a turn-based event. You and the Snorlax take turns. On your turn you throw a Pokéball which can hit (catch the Snorlax) or miss. On Snorlax's turn it can run away or stay. The encounter continues until the Snorlax runs away, or you catch the Snorlax.

   - The probability that you throw a Pokéball which hits (catches) the Snorlax is 0.7.
   - The probability that the Snorlax runs away on its turn is 0.6.

   Suppose that:

   - Snorlax is a slow Pokémon, so you have the first turn. You alternate thereafter.
   - You have an infinite number of Pokéballs to throw.
   - Each turn is independent.

   What is the probability that you catch the Snorlax? (You throw a successful Pokéball before the Snorlax runs.)

H

$\underline{H'R'}$ H

$\underline{H'R'}\ \underline{H'R'}$ H

$\vdots$

$$\sum_{k=0}^{\infty}\left((0.3)(0.4)\right)^k 0.7 = \frac{0.7}{1-0.12} =$$

3. 0.7955

B

4. (5 points) Professor Oak claims that the encounter rates for the Pokémon found in Mt. Moon are as follows:

Zubat	0.40
Geodude	0.30
Paras	0.20
Clefairy	0.10

   Suppose you encounter six Pokémon on a journey through Mt. Moon. Assuming Professor Oak is correct, what is the probability that you encounter 1 Zubat, 2 Geodude, 2 Paras, and 1 Clefairy?

$$\frac{6!}{1!\,2!\,2!\,1!}(0.4)^1(0.3)^2(0.20)^2(0.1)^1 =$$

4. 0.02592

B

5. (13 points) Suppose that grades on the previous semester's STAT 400 Exam II were not very good. Graphed, their distribution had a shape similar to the probability density function

$$f(g) = \frac{1}{8525}(2g+10), \quad 45 \leq g \leq 100.$$

Assume that scores on this exam, G, actually follow this distribution.

(a) (5 points) Calculate the probability that a randomly chosen student scores below an 80?

$$P[G<80] = \int_{80}^{100} \frac{1}{8525}(2g+10)\,dg =$$

(a) 0.5543

(b) (5 points) Calculate the mean (expected value) of the grades, E[G].

$$E[G] = \int_{45}^{100} g \frac{1}{8525}(2g+10)\,dg =$$

(b) 75.75

(c) (3 points) Write a single integral expression that would evaluate to the variance of the grades.

DEFINE $\mu = 75.75$

$$\int_{45}^{100} (g-\mu)^2 \frac{1}{8525}(2g+10)\,dg$$

(c)

B

6. (14 points) Suppose that amount of caffeine in a small (10oz) CoffeeBucks coffee follows a normal distribution with a mean of 220 mg and a standard deviation of 25 mg.

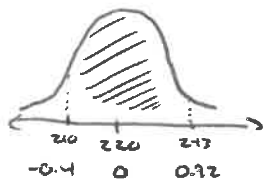
(a) (4 points) Calculate the probability that a small CoffeeBucks coffee contains between 210 and 243 mg of caffeine.

$$P(210 < CB < 243) = P(-0.4 < Z < 0.92)$$

$$= P(Z < 0.92) - P(Z < -0.4)$$

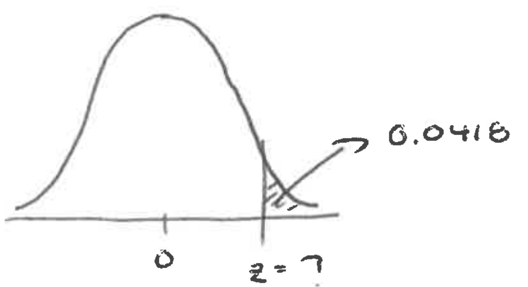
$$\frac{210-220}{25} = -0.4 \qquad = 0.8212 - 0.3446 =$$

$$\frac{243-220}{25} = 0.92$$



(a) 0.4766

(b) (4 points) How much caffeine must a small CoffeeBucks coffee contains to be in the top 4.18% of caffeine content of CoffeeBucks coffees?



$z = 1.73$

$c = 220 + (1.73)(25) =$

(b) 263.25

(c) (6 points) Dr. Elliot Reid was on call last night at Sacred Heart Hospital. To get ready for morning rounds, she takes a 150 mg caffeine pill, then drinks a large (20oz) CoffeeBucks coffee, which contains twice the caffeine of a small (10oz) CoffeeBucks coffee. What is the probability that she just ingested more than 600 mg of caffeine?

$\mu = 150 + 2(220) = 590$

$\sigma^2 = 2^2(25)^2 = 50^2$

$\sigma = 50$

$$P(C > 600) = P(Z > 0.2)$$

$$= P(Z < -0.2) =$$

(c) 0.4207

C

3. (14 points) Every morning David uses the CUMTD buses to get to work. After walking to the nearest bus stop, he boards the first bus to arrive, which is a `9 Brown` 60% of the time, a `14 Navy` 30% of the time, or a `1 Yellow` the other 10% of the time. By riding these three buses for years, David has determined that there is a 4% chance he is late to work if he rides a `9 Brown`, a 30% chance he is late to work if he rides a `14 Navy`, and a 80% chance he is late if instead he takes a `1 Yellow`.

   (a) (4 points) Calculate the probability that David is late to work.

$P(B \cap L) = P(B)\,P(L|B) = (0.60)(0.04) = 0.024$

	L	OT	
B	0.024	0.576	0.60
N	0.09		0.3
Y	0.08		0.1
	0.194	0.806	1

$P(\text{LATE}) = P(B \cap L) + P(N \cap L) + P(Y \cap L)$

(a) 0.194

   (b) (6 points) Suppose David is on time to work. What is the probability that he took the `9 Brown` that morning?

$P(B|OT) = \dfrac{P(B \cap OT)}{P(OT)} = \dfrac{0.576}{0.806} =$

(b) 0.7146

   (c) (4 points) David really dislikes riding the `1 Yellow`. David is also not a fan of being late to work. What is the probability that David's day has a rough start? (That is, what is the probability that David is late, or rode the `1 Yellow`, or both?)

$P(L \cup Y) = P(L) + P(Y) - P(L \cap Y)$

$= 0.194 + 0.1 - 0.08 =$

(c) 0.214

C

5. (12 points) When Alexandra plays chess against her favorite computer program, she wins with probability 0.7, loses with probability 0.2, and her games result in a draw with probability 0.1. Assume games are independent.

(a) (4 points) If Alexandra plays five games, calculate the probability that she wins at least four of the games.

$X = \#$ wins $\quad X \sim b(n=5, p=0.7)$

$P[X \geq 4] = P[X=4] + P[X=5]$

$= \binom{5}{4}(0.7)^4(0.3)^1 + \binom{5}{5}(0.7)^5(0.3)^0 =$

(a) 0.5282

(b) (4 points) If Alexandra plays ten games, calculate the probability that she draws at least one of the games.

$Y = \#$ DRAWS $\quad Y \sim b(n=10, p=0.1)$

$P[Y \geq 1] = 1 - P[Y=0]$

$= 1 - \binom{10}{0}(0.1)^0(0.9)^{10} =$

(b) 0.6513

(c) (4 points) If Alexandra plays until she loses, calculate the probability that her first loss happens anytime after the third game.

$Z = \#$ TRIALS $\quad Z \sim Geom(p=0.2)$

$P[Z > 3] = (1-0.2)^3 =$

(c) 0.512

C

9. (14 points) On days that he does not have his morning coffee, the number of mistakes Professor Fisher makes during a lecture follows a Poisson distribution with an average of 3.2 per lecture. On days he has his morning coffee, the distribution of mistakes again follows a Poisson distribution, but with a reduced average of 2.1 per lecture. There is a 60% chance that Professor Fisher remembers to drink his morning coffee each day.

(a) (4 points) Assuming Professor Fisher had his morning coffee, what is the probability that he makes at most two mistakes in a lecture?

$\lambda = 2.1$ / LECTURE WITH COFFEE

Y = # MISTAKES $\quad Y \sim Pois(\lambda = 2.1)$

$$P[Y \le 2] = P[Y=0] + P[Y=1] + P[Y=2]$$

$$= \frac{(2.1)^0 e^{-2.1}}{0!} + \frac{(2.1)^1 e^{-2.1}}{1!} + \frac{(2.1)^2 e^{-2.1}}{2!} =$$

(a) 0.6496

(b) (4 points) Assuming Professor Fisher did not have his morning coffee, what is the probability that he makes two mistakes in the first half of a lecture?

$\lambda = \frac{3.2}{2} = 1.6$ / HALF LECTURE WITHOUT COFFEE

X = # MISTAKES $\quad X \sim Pois(\lambda = 1.6)$

$$P[X = 2] = \frac{(1.6)^2 e^{-1.6}}{2!} =$$

(b) 0.2584

(c) (6 points) You attend a lecture and Professor Fisher manages to not make any mistakes! What is the probability that he had his morning coffee?

$$P[COF \mid m=0] = \frac{P[COF]P[m=0 \mid COF]}{P[COF]P[m=0 \mid COF] + P[COF']P[m=0 \mid COF']}$$

$$= \frac{0.6e^{-2.1}}{0.6e^{-2.1} + 0.4e^{-3.2}} =$$

(c) 0.8183