STAT 400 Homework 05

Spring 2018 | Dalpiaz | UIUC

***Due:** Friday, February 23, 2:00 PM*

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Consider a random variable X with the probability mass function

$$f(x) = \frac{6}{3^x}, \quad x = 2, 3, 4, 5, \ldots$$

(a) Find the moment-generating function of X, $M_X(t)$. Report the function, being sure to indicate the values of t where the function exists.

(b) Calculate $\mathrm{E}[X]$.

Exercise 2

How much wood would a woodchuck chuck if a woodchuck could chuck wood? Let W denote the amount of wood a woodchuck would chuck per day (in cubic meters) if a woodchuck could chuck wood. Suppose the moment-generating function of W is

$$M_W(t) = 0.1 \cdot e^{3t} + 0.3 \cdot e^{2t} + 0.5 \cdot e^{1t} + 0.1.$$

(a) Calculate the average amount of wood a woodchuck would chuck per day, $\mathrm{E}[W]$.

(b) Calculate $\mathrm{Var}[W]$.

Exercise 3

Consider a random variable Y with the probability density function

$$f(y) = \frac{|y|}{5}, \quad -1 < y < 3.$$

(a) Calculate $\mathrm{E}[Y]$.

(b) Calculate $\mathrm{median}[Y]$, the median of Y.

Exercise 4

Suppose that scores on the previous semester's STAT 400 Exam II were not very good. Graphed, their distribution had a shape similar to the probability density function

$$f(s) = \frac{1}{9000}(2s + 10), \quad 40 \le s \le 100.$$

Assume that scores on this exam, S, actually follow this distribution. (Note: This distribution does not necessarily reflect reality.)

(a) Suppose 10 students from the class are selected at random. What is the probability that (exactly) 4 of them received a score above 85?

(b) What was the standard deviation of the scores, $\mathrm{SD}[S]$?

(c) What was the class 40th percentile? That is, find a such that $P(S \le a) = 0.40$.

Exercise 5

Students often worry about the time it takes to complete an exam. Suppose that completion time in hours, T, for the STAT 400 final exam follows a distribution with density

$$f(t) = \frac{2}{27}(t^2 + t), \quad 0 \le t \le 3.$$

What is the probability that a randomly chosen student finishes the exam during the second hour of the exam. That is, calculate $P(1 < T < 2)$.