STAT 400 Homework 06

Spring 2018 | Dalpiaz | UIUC

Due: *Friday, March 9, 2:00 PM*

Please see the **detailed homework policy document** for information about homework formatting, submission, and grading.

Exercise 1

Consider a random variable X with the moment generating function

$$M_X(t) = e^{5t+8t^2} = \exp(5t + 8t^2)$$

(a) Calculate $P(4 < X < 16)$.

(b) Calculate $P(4 < X^2 < 16)$.

Exercise 2

Consider a random variable X with $\mathrm{E}[X] = 5$ and $\mathrm{Var}[X] = 16$.

(a) Calculate $P(|x - 5| < 6)$ if X follows a normal distribution.

(b) Use Chebyshev's inequality to provide a lower bound for $P(|x - 5| < 6)$. (No longer assume X is normal.)

Exercise 3

In the original Pokémon Red and Blue, there were 151 Pokémon, but only 150 of these Pokémon could actually be caught or obtained in the games. If you wanted to "catch 'em all", the 151st Pokémon, Mew, could only be obtained through special giveaway events at local video game retailers. (For example FuncoLand, which was later purchased by GameStop.)

Suppose that these giveaway events at your local FuncoLand occur according to a Poisson process with an average of one event per two months.

For this exercise, assume that all 12 months of the year have the same number of days. Additionally, assume that the four seasons all last exactly three months.

- Winter: December, January, February
- Spring: March, April, May
- Summer: June, July, August
- Fall: September, October, November

Also, suppose that it is the beginning of a new year, that is January 1, 1999.

(a) What is the probability that the first event occurs before Spring?

(b) What is the probability that the first event occurs during the month of March?

(c) What is the probability that the third event occurs during Summer?

(d) What is the probability that the fifth event occurs before the end of the year?

Exercise 4

In Neverland, annual income (in $), X, is distributed according to a Gamma distribution with $\alpha = 5$ and $\theta = 10,000$. Every year, the IRS audits 1% of the individuals with an income below $50,000$, 3% of individuals with incomes between $50,000$ and $95,000$, and 6% of individuals with an income above $95,000$. Suppose that the individuals to be audited are selected at random.

(a) What is the distribution of the income groups? That is, what proportion of Neverland's population falls into each of the three income groups? Even more specifically, find $P(X < \$50,000)$, $P(\$50,000 < X < \$95,000)$, and $P(X > \$95,000)$.

- Hint: These probabilities should add to 1.

(b) You overhear Mr. Statman complain about being audited. What is the probability that Mr. Statman's income is below $50,000$? Between $50,000$ and $95,000$? Above $95,000$?

- Hint: Again, these probabilities should add to 1. Essentially, we're finding the (posterior) distribution of Mr. Statman's possible income (group) given that he's being audited.