

STAT 400 Homework 11

Spring 2018 / Dalpiaz / UIUC

Due: Friday, April 27, 2:00 PM

Exercise 1

Before it closed, Ron Swanson was a frequent patron of Charles Mulligan's Steakhouse in Indianapolis, Indiana. Ron enjoyed the experience so much, during each visit he took a picture with his steak.



Ron also weighed each steak he consumed. He has a record of eating six “22 ounce” Charles Mulligan's porterhouse steaks. Ron found that these six steaks weighed

22.4 oz, 20.8 oz, 21.6 oz, 20.2 oz, 21.4 oz, 22.0 oz

Suppose that “22 ounce” Charles Mulligan's porterhouse steaks follow a $N(\mu, \sigma^2)$ distribution and that Ron's six steaks were a random sample.

Recall that

$$\bar{x} = 21.4 \quad \text{and} \quad s^2 = 0.64.$$

(a) Calculate the test statistic for testing $H_0 : \mu = 22$, where μ is the true weight of a “22 ounce” Charles Mulligan's porterhouse. Report your answer rounded to three decimal places.

Solution:

- $\bar{x} = 21.4$
- $\mu_0 = 22$
- $n = 6$
- $s = 0.8$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{21.4 - 22}{0.8/\sqrt{6}} = \boxed{-1.837}$$

(b) Calculate the p-value for the test $H_0 : \mu = 22$ versus $H_1 : \mu < 22$. Report your decision when $\alpha = 0.05$.

Some useful code:

- Excel: T.DIST(x, df, 1) gives the area to the **left** of x.
- Excel: TDIST(x, df, 1) gives the area to the **right** of x. (An older command. Only accepts positive values for x.)
- R: pt(x, df) gives the area to the **left** of x.

Solution:

This is a one-sided test, with a less-than alternative, so

$$\text{p-value} = P(T_5 < -1.837) = \boxed{0.06282}$$

$$\text{p-value} = 0.06282 > \alpha = 0.05$$

Fail to reject H_0 .

```
pt(q = -1.837, df = 5)
```

```
## [1] 0.06281569
```

(c) Calculate the test statistic for testing $H_0 : \sigma \leq 0.50$, where σ is the true standard deviation of the weight of a “22 ounce” Charles Mulligan’s porterhouse.

Solution:

- $\sigma_0 = 0.50$
- $n = 6$
- $s = 0.8$

$$X^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(6-1)0.8^2}{0.50^2} = \boxed{12.8}$$

(d) Calculate the p-value for the test $H_0 : \sigma \leq 0.50$ versus $H_1 : \sigma > 0.50$. Report your decision when $\alpha = 0.01$.

Some useful code:

- Excel: CHISQ.DIST(x, df, 1) gives the area to the **left** of x.
- Excel: CHIDIST(x, df, 1) gives the area to the **right** of x. (An older command.)
- R: pchisq(x, df) gives the area to the **left** of x.

Solution:

This is a one-sided test, with a greater-than alternative, so

$$\text{p-value} = P(\chi_5^2 > 12.8) = \boxed{0.02533}$$

$$\text{p-value} = 0.02533 > \alpha = 0.01$$

Fail to reject H_0 .

```
pchisq(12.8, df = 5, lower.tail = FALSE)
```

```
## [1] 0.02532684
```

(e) Suppose the true value of σ is indeed 0.50. Was your decision in part (d) correct? If not, what Type of error was made? Justify your answer.

Solution:

If $\sigma = 0.50$, then H_0 is true. By failing to reject, we have made the correct decision. Had we rejected, we would have made an error, in particular, a Type I Error.

Exercise 2

Last year, ballots in Champaign-Urbana contained the following question to assess public opinion on an issue:

“Should the State of Illinois legalize and regulate the sale and use of marijuana in a similar fashion as the State of Colorado?”

Suppose that we would like to understand Champaign-Urbana’s 2017 opinion on marijuana legalization. To satisfy our curiosity, we obtain a random sample of 120 Champaign-Urbanians and find that 87 support marijuana legalization.

(a) Calculate the z test statistic for the test $H_0 : p = 0.70$ versus $H_1 : p > 0.70$ where p is the true proportion of Champaign-Urbanians that support marijuana legalization. Report your answer rounded to two decimal places.

Solution:

- $x = 87$
- $n = 120$

$$\hat{p} = \frac{x}{n} = \frac{87}{120} = 0.725$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.725 - 0.70}{\sqrt{\frac{0.70 \cdot 0.30}{120}}} \approx \boxed{0.60}$$

```
(0.725 - 0.70) / sqrt(0.70 * 0.30 / 120)
```

```
## [1] 0.5976143
```

(b) Calculate the (approximate, using z) p-value for the test $H_0 : p = 0.70$ versus $H_1 : p > 0.70$.

Solution:

This is a one-sided test, with a greater-than alternative, so

$$\text{p-value} = P(Z > 0.60) = P(Z < -0.60) = \boxed{0.2743}$$

```
z = (0.725 - 0.70) / sqrt(0.70 * 0.30 / 120)
pnorm(q = z, mean = 0, sd = 1, lower.tail = FALSE) # some rounding differences
```

```
## [1] 0.2750487
```

(c) Calculate the **exact** p-value for the test $H_0 : p = 0.70$ versus $H_1 : p > 0.70$. That is, calculate the probability of seeing as many supporters as observed, or more, assuming the null hypothesis is true.

Some useful code:

- Excel: `BINOM.DIST(x, n, p, 1)` gives the probability of less than or equal to x .
- R: `pnbinom(x, n, p)` gives the probability of less than or equal to x .

Solution:

Here,

$$X \sim \text{binom}(n = 120, p = 0.70).$$

Then,

$$\text{p-value} = P(X \geq 87) = 1 - P(X \leq 86) = \boxed{0.3129}$$

```
1 - pbinom(q = 86, size = 120, prob = 0.70)
## [1] 0.3128955
pbinom(q = 86, size = 120, prob = 0.70, lower.tail = FALSE)
## [1] 0.3128955
```

Exercise 3

Last year, ballots in Champaign-Urbana contained the following question to assess public opinion on an issue:

“Should the State of Illinois legalize and regulate the sale and use of marijuana in a similar fashion as the State of Colorado?”

Suppose we obtain a random sample of 80 Champaign voters, of which 55 support marijuana legalization. We also obtain a random sample of 100 Urbana voters, of which 75 support marijuana legalization. Let p_C be the true proportion of Champaign voters who support marijuana legalization and let p_U be the true proportion of Urbana voters who support marijuana legalization.

(a) Calculate a 99% confidence interval for $p_U - p_C$.

Solution:

- $x_C = 55$
- $n_C = 80$
- $x_U = 75$
- $n_U = 100$
- $z_{\alpha/2} = z_{0.005} = 2.576$

$$\hat{p}_C = \frac{x_C}{n_C} = \frac{55}{80} = 0.6875$$

$$\hat{p}_U = \frac{x_U}{n_U} = \frac{75}{100} = 0.75$$

$$(\hat{p}_U - \hat{p}_C) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_C(1 - \hat{p}_C)}{n_C} + \frac{\hat{p}_U(1 - \hat{p}_U)}{n_U}}$$

$$(0.75 - 0.6875) \pm 2.576 \sqrt{\frac{0.6875(1 - 0.6875)}{80} + \frac{(0.75)(1 - 0.75)}{100}}$$

$$\boxed{0.0625 \pm 0.1740}$$

(b) Calculate the p-value for the test $H_0 : p_U = p_C$ versus $H_1 : p_U \neq p_C$.

Solution:

We first need to calculate the pooled proportion estimate under the null hypothesis.

$$\hat{p} = \frac{x_C + x_U}{n_C + n_U} = \frac{55 + 75}{80 + 100} = \frac{13}{18}$$

We then calculate the test statistic.

$$z = \frac{(\hat{p}_U - \hat{p}_C) - 0}{\sqrt{\hat{p}(1 - \hat{p})(\frac{1}{n_C} + \frac{1}{n_U})}} = \frac{(0.75 - 0.6875) - 0}{\sqrt{(\frac{13}{18})(1 - \frac{13}{18})(\frac{1}{80} + \frac{1}{100})}} = 0.9303$$

Since this is a two-sided test, we have,

$$\text{p-value} = 2 \cdot P[Z > 0.9303] = 2 \cdot P[Z < -0.9303] = 2 \cdot 0.1761 = \boxed{0.3522}.$$