STAT 400
UIUC

Practice Problems #10

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The following are a number of practice problems that may be *helpful* for completing the homework, and will likely be **very useful** for studying for exams.

1 – 3. A store sells "16-ounce" boxes of *Captain Crisp* cereal. A random sample of 9 boxes was taken and weighed. The results were the following (in ounces):

15.5 16.2 16.1 15.8 15.6 16.0 15.8 15.9 16.2

Assume the weight of cereal in a box is normally distributed.

1. a) Compute the sample mean $\bar{x}$ and the sample standard deviation s.

Do NOT use a computer. You may only use +, –, ×, ÷, and $\sqrt{\ }$ on a calculator.

Show ALL work.

2. b) Construct a 95% confidence interval for the overall average weight of boxes of *Captain Crisp* cereal.

c) Construct a 95% confidence upper bound for the overall average weight of boxes of *Captain Crisp* cereal.

d) Construct a 90% confidence lower bound for the overall average weight of boxes of *Captain Crisp* cereal.

3. e) Construct a 90% confidence interval for the overall standard deviation of the weights of boxes of *Captain Crisp* cereal.

~~f) Construct the 90% minimum length confidence interval for the overall standard deviation of the weights of boxes of *Captain Crisp* cereal.~~

g) Construct a 99% confidence lower bound for the overall standard deviation of the weights of boxes of *Captain Crisp* cereal.

4. a) What is the minimum sample size required to estimate the overall mean weight of boxes of *Captain Crisp* cereal to within 0.01 ounces with 95% confidence, if the overall (population) standard deviation of the weights is 0.20 ounces?

b) What is the minimum sample size required to estimate the overall proportion of boxes that have less than 16 ounces of cereal to within 2% with 95% confidence, if no guess as to the value of this proportion is available?

c) What is the minimum sample size required to estimate the overall proportion of boxes that have less than 16 ounces of cereal to within 2% with 95% confidence, if it is known that this proportion is at most 0.30?

5 – 6. A random sample of 144 patients, suffering from a particular disease are given a new medicine. 108 of the patients report an improvement in their condition.

5. a) Construct a 90% (two-sided) confidence interval for the overall improvement rate of this medicine.

b) Find the minimum sample size required if we want to estimate the improvement rate of this medicine to within 4% with 90% confidence if it is known that the improvement rate is between 70% and 90%.

c) Find the minimum sample size required if we want to estimate the improvement rate of this medicine to within 4% with 90% confidence if we do not make any assumptions about the improvement rate.

6. d) Construct a 90% confidence lower bound for the overall improvement rate.

e) Construct a 95% confidence upper bound for the overall improvement rate.

f) The company that manufactures this medicine claims an 80% improvement rate. Based on your answer to part (e), do you find this claim believable? Explain your answer.

7. The United States Secret Service (USSS) hires you to compute a 95% confidence interval for the overall average maximum weight (in pounds) its agents can bench press in a 5-repetition set. Stressing the top secret nature of the data, the USSS does not make the data available to you, it does not even report the values of the sample mean and the sample standard deviation. The only information you have is that an 80% confidence interval based on a sample of 14 agents is $(250, 277)$. Assume the population is approximately normally distributed. Compute a 95% confidence interval for the overall average maximum weight USSS agents can bench press in a 5-repetition set.

8 – 10. A random sample of size $n = 289$ from a $N(\mu, \sigma^2)$ distribution gives the sample mean $\bar{x} = 789$ and the sample standard deviation $s = 34$.

8. If the number of degrees of freedom (d.f.) is large, the values of $t_{\alpha/2}$ can be approximated by $z_{\alpha/2}$.

a) Compare $z_{0.05} = 1.645$ to the value of $t_{0.05}(n-1 \text{ d.f.})$.

"Hint": EXCEL =TINV(two-tail probability , degrees of freedom).

That is, =TINV(α , $n - 1$).

OR R `> qt(` probability to the left `,` degrees of freedom `)`.

That is, `> qt(` $1 - \alpha/2$ `,` $n - 1$ `)`.

b) Construct a 90% (two-sided) confidence interval for μ.

c) Compare $z_{0.025} = 1.96$ to the value of $t_{0.025}(n-1 \text{ d.f.})$.

d) Construct a 95% (two-sided) confidence interval for μ.

9. If the number of degrees of freedom (d.f.) is large, the values of chi-squared distribution can be approximated by the values of a Normal distribution with mean $\mu = \text{d.f.}$ and variance $\sigma^2 = 2 \times \text{d.f.}$

a) Find the two values of a Normal distribution $\mathrm{N}(\mu = \text{d.f.}, \sigma^2 = 2 \times \text{d.f.})$ distribution with area 0.05 to the left and to the right, respectively. Here, $\text{d.f.} = n - 1$.

b) Compare the answers from part (a) with the values of $\chi^2_{0.95}(n - 1 \text{ d.f.})$ and $\chi^2_{0.05}(n - 1 \text{ d.f.})$.

"Hint": EXCEL `=CHIINV(` probability to the right , degrees of freedom `)`.

OR R `> qchisq(` probability to the left `,` degrees of freedom `)`.

c) Construct a 90% (two-sided) confidence interval for σ.

10. 9. (continued)

d) Find the probability $\mathrm{P}(X > 312)$ using …

i) … $\mathrm{N}(\mu = 288, \sigma^2 = 2 \times 288)$ distribution;

ii) … $\chi^2(288 \text{ d.f.})$ distribution.

"Hint": EXCEL `=CHIDIST(` x , degrees of freedom `)` gives area to the *right* of x.

OR R `> pchisq(` x `,` degrees of freedom `)` gives area to the *left* of x.

e) Find the probability $\mathrm{P}(270 < X < 300)$ using …

i) … $\mathrm{N}(\mu = 288, \sigma^2 = 2 \times 288)$ distribution;

ii) … $\chi^2(288 \text{ d.f.})$ distribution.