

General Probability, I: Rules of probability

Some basic probability rules

1. Kolmogorov axioms:

- (1) Total probability 1: $P(S) = 1$
- (2) Nonnegativity: $P(A) \geq 0$
- (3) Addition formula: If A_1, A_2, \dots are disjoint, then $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$.

2. Complement rule: $P(A') = 1 - P(A)$.

3. Empty set: $P(\emptyset) = 0$.

4. Union of two events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5. Union of three events (inclusion/exclusion formula):

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

Notes and tips

- **Draw Venn diagrams:** Venn diagrams help you picture what is going on and deriving the appropriate probabilities.
- **Probabilities behave like areas:** A very useful rule of thumb is that probabilities behave like areas in the associated Venn diagrams. All of the above formulas are consistent with this rule (assuming areas are normalized so that the entire sample space S has area 1), and you can derive each of these rules simply by calculating areas in Venn diagrams. Moreover, the area trick makes it very easy to find probabilities in other, more complicated, situations. For example, the probability that exactly one of A, B, C occurs corresponds to the area of those parts of $A, B,$ and C in the corresponding Venn diagram that don't overlap with any of the other sets.
- **Use proper notation and distinguish between a set, A , and its probability $P(A)$:** Writing $A \cup B = 0.9$ is nonsensical since $A \cup B$ is a set and cannot equal a number. The correct notation is $P(A \cup B) = 0.9$. Similarly, $A \cup B = A + B - A \cap B$ is wrong; the equality should be between probabilities, not sets, i.e., $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Remember that arithmetic operations ($+$, $-$, etc.) between sets don't make sense; when dealing with sets themselves (rather than their probabilities), you must use set-theoretic notation (\cup , \cap , \setminus , etc.).