STAT 400: Point Estimation

Dalpiaz, Stepanov

Maximum Likelihood Estimation (MLE)

The following procedure will be used in STAT 400 to to calculate the maximum likelihood estimator for a single parameter using a independent and identically distributed random sample.

Consider an iid random sample X_1, X_2, ..., X_n from a particular distribution $f(x; \theta)$ with a single parameter θ.

"Multiply." **Step 1:** Write out the likelihood function:

$$L(\theta) = \prod_{i=1}^{n} f(x_i; \theta)$$

"Simplify." **Step 2:** Frequently it will help to simplify this expression algebraically.

"Take a log." **Step 3:** Finding the maximum of the log of the likelihood is often easier.

$$\ln(L(\theta))$$

"Take a derivative." **Step 4:** How do you find the maximum? Take a derivative with respect to the parameter of interest. (And maybe some other stuff, but we won't worry about that.)

$$\frac{d}{d\theta} \ln(L(\theta))$$

"Set equal to zero." **Step 5:** The second part of finding a maximum. (Again, we're ignoring some things.)

$$\frac{d}{d\theta} \ln(L(\theta)) = 0.$$

"Solve." **Step 6:** Solve for θ.

"Draw a hat." **Step 7:** Put a hat over θ. Congratulations. You found the MLE!

"Verify." **Step 8:** Does your estimator make sense? Does the estimator output values in the parameter space given inputs in the sample space?

This procedure is not garuanteed to work in general, but will work for any distribution used on homework and exams in STAT 400. See STAT 410 and STAT 510 for details on exceptions to this procedure, that is, when maximizing the likelihood is more difficult.

Method of Moments (MoM)

The following procedure will be used in STAT 400 to to calculate the methof of moments estimator for a single parameter using a independent and identically distributed random sample.

Consider an iid random sample X_1, X_2, ..., X_n from a particular distribution $f(x;\theta)$ with a single parameter θ.

"Find." **Step 1:** Find $E[X]$, the population mean.

Maybe this is a common distribution and we already know E$[X]$? ☺

Maybe it's not and we have to calculate an integral? ☹

Either way we will have some expression in terms of θ, which we will call $g(\theta)$. In other words, find $E[X] = g(\theta)$.

"Equate." **Step 2:** Set $g(\theta)$, the population mean, equal to $\bar{X}$, the sample mean.

"Solve." **Step 3:** Solve for θ.

"Draw a tilde." **Step 4:** Put a tilde over θ. Congratulations. You found the MoM estimator!

"Verify." **Step 5:** Does your estimator make sense? Does the estimator output values in the parameter space given inputs in the sample space? (This can happen with MoM, but if it does, should you use MoM?)