STAT 400
UIUC

Examples for 1.1

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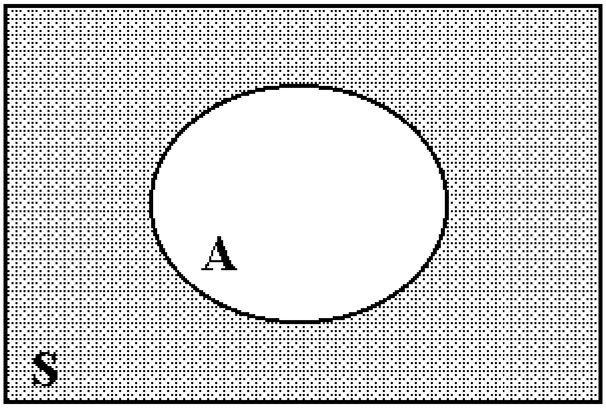
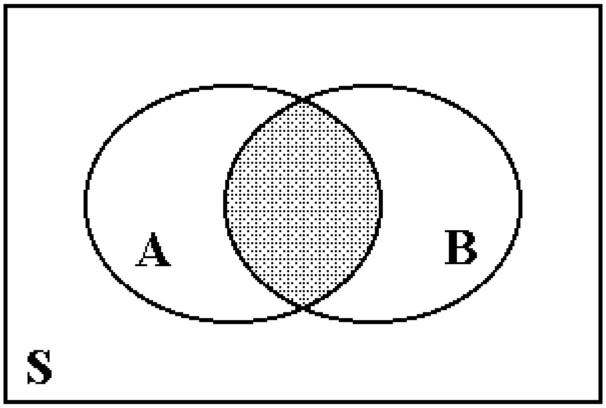


Complement of A

$$A'$$

(not A, $\overline{A}$, A^c)

contains all elements
that are **not** in A

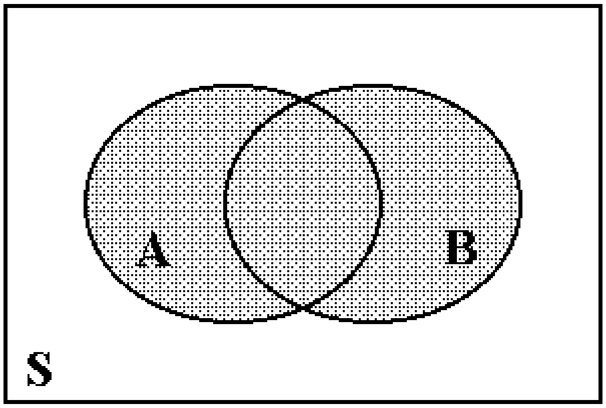


Intersection of A and B

$$A \cap B$$

(A and B, A B)

contains all elements
that are in A **and** in B



Union of A and B

$$A \cup B$$

(A or B)

contains all elements
that are either in A **or** in B
or both

Axiom 1 Let A be any event defined over S. Then $P(A) \ge 0$.

Axiom 2 $P(S) = 1$.

Axiom 3 If $A_1, A_2, A_3, \ldots$ are events and $A_i \cap A_j = \varnothing$ for each $i \ne j$, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_k) = P(A_1) + P(A_2) + \ldots + P(A_k)$$

for each positive integer k, and

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$$

for an infinite, but countable, number of events.

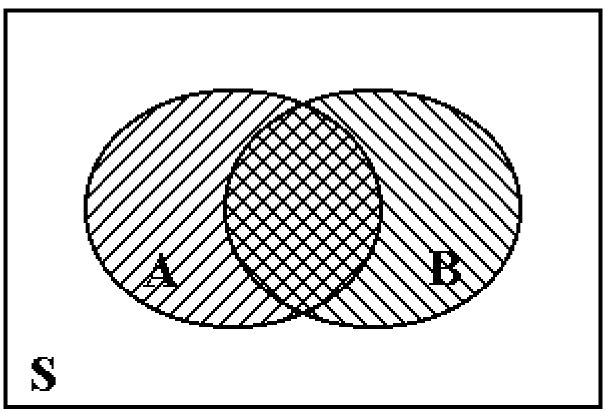
Theorem 1. $P(A') = 1 - P(A)$.

Theorem 2. $P(\varnothing) = 0$.

Theorem 3. If $A \subset B$, then $P(A) \leq P(B)$.

Theorem 4. For any event A, $P(A) \leq 1$.

!	For any event A, $0 \leq P(A) \leq 1$	!	$P(S) = 1$, where S is the sample space.	!



Theorem 5.

If A and B are any two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Theorem 6.

$$\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & + P(A \cap B \cap C) \end{aligned}$$

$$\begin{aligned} P(A \cup B \cup C \cup D) = {} & P(A) + P(B) + P(C) + P(D) \\ & - P(A \cap B) - P(A \cap C) - P(A \cap D) \\ & - P(B \cap C) - P(B \cap D) - P(C \cap D) \\ & + P(A \cap B \cap C) + P(A \cap B \cap D) \\ & + P(A \cap C \cap D) + P(B \cap C \cap D) \\ & - P(A \cap B \cap C \cap D) \end{aligned}$$

• • •

1. Suppose a 6-sided die is rolled. The sample space, S, is $\{1, 2, 3, 4, 5, 6\}$. Consider the following events:

$A = \{$ the outcome is even $\}$,

$B = \{$ the outcome is greater than 3 $\}$,

a) List outcomes in A, B, A', $A \cap B$, $A \cup B$.

b) Find the probabilities $P(A)$, $P(B)$, $P(A')$, $P(A \cap B)$, $P(A \cup B)$ if the die is balanced (fair).

c) Suppose the die is loaded so that the probability of an outcome is proportional to the outcome, i.e.

$$P(1) = p, \quad P(2) = 2p, \quad P(3) = 3p, \quad P(4) = 4p, \quad P(5) = 5p, \quad P(6) = 6p.$$

i) Find the value of p that would make this a valid probability model.

ii) Find the probabilities $P(A)$, $P(B)$, $P(A')$, $P(A \cap B)$, $P(A \cup B)$.

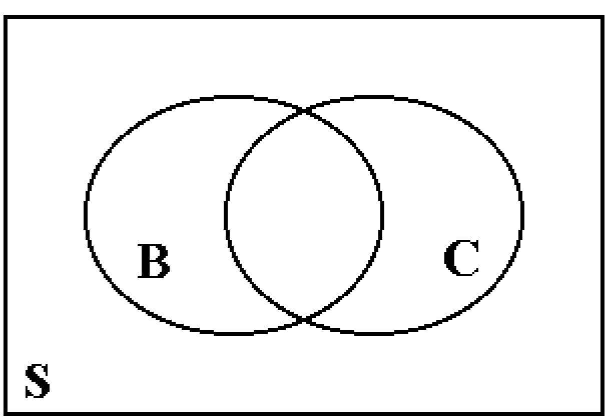
2. Consider a “thick” coin with three possible outcomes of a toss (Heads, Tails, and Edge) for which Heads and Tails are equally likely, but Heads is five times as likely than Edge. What is the probability of Heads ?

3. The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

a) What is the probability that a student selected at random does not own a bicycle?

b) What is the probability that a student selected at random owns either a car or a bicycle, or both?

c) What is the probability that a student selected at random has neither a car nor a bicycle?



	C	C′	
B			
B′			

4. During the first week of the semester, 80% of customers at a local convenience store bought either beer or potato chips (or both). 60% bought potato chips. 30% of the customers bought both beer and potato chips. What proportion of customers bought beer?

5. Suppose

$P(A) = 0.22$,

$P(B) = 0.25$,

$P(C) = 0.28$,

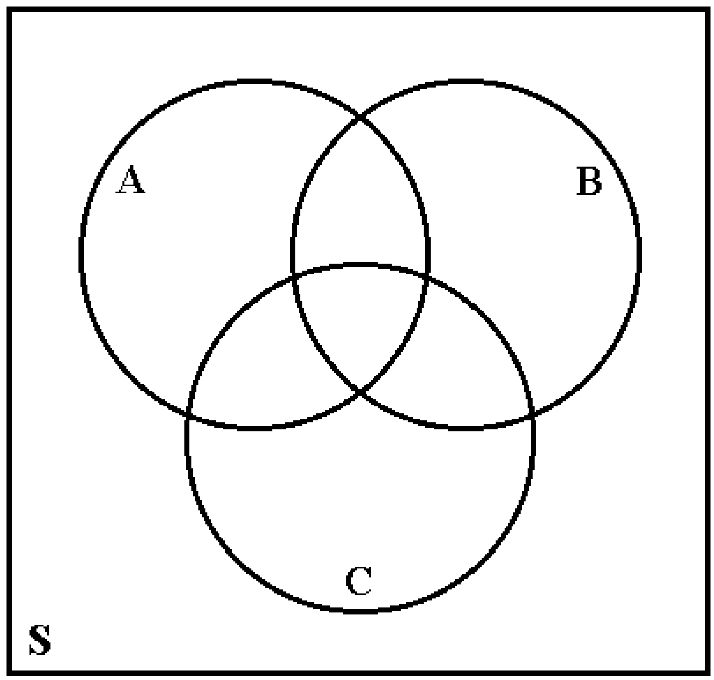
$P(A \cap B) = 0.11$,

$P(A \cap C) = 0.05$,

$P(B \cap C) = 0.07$,

$P(A \cap B \cap C) = 0.01$.

Find the following:



a) $P(A \cup B)$

b) $P(A' \cap B')$

c) $P(A \cup B \cup C)$

d) $P(A' \cap B' \cap C')$

e) $P(A' \cap B' \cap C)$

f) $P((A' \cap B') \cup C)$

g) $P((A \cup B) \cap C)$

h) $P((B \cap C') \cup A')$

6. Let $a > 2$. Suppose S = $\{ 0, 1, 2, 3, \dots \}$ and

$$P(0) = c, \qquad P(k) = \frac{1}{a^k}, \quad k = 1, 2, 3, \dots .$$

a) Find the value of c (c will depend on a) that makes this is a valid probability distribution.

b) Find the probability of an odd outcome.

7. Suppose S = $\{ 0, 1, 2, 3, \dots \}$ and

$$P(0) = p, \qquad P(k) = \frac{1}{2^k \cdot k!}, \quad k = 1, 2, 3, \dots .$$

Find the value of p that would make this a valid probability model.