Examples for 2.1, 2.2, 2.3 (2)

Stepanov
Dalpiaz

1. Homer Simpson is going to Moe's Tavern for some *Flaming Moe*'s. Let X denote the number of *Flaming Moe*'s that Homer Simpson will drink. Suppose X has the following probability distribution:

x	$f(x)$
0	0.1
1	0.2
2	0.3
3	0.3
4	

a) Find the missing probability $f(4) = P(X = 4)$.

b) Find the probability $P(X \geq 1)$.

c) Find the probability $P(X \geq 1 \mid X < 3)$.

d) Compute the expected value of X, $E(X)$.

e) Compute the standard deviation of X, $SD(X)$.

1. (continued)

Suppose each *Flaming Moe* costs \$1.50, and there is a cover charge of \$1.00 at the door. Let Y denote the amount of money Homer Simpson spends at the bar. Then $Y = 1.50 \cdot X + 1.00$.

f) Find the probability that Homer would spend over \$5.00.

g) Find the expected amount of money that Homer Simpson would spend, $E(Y)$.

h) Find the standard deviation for the amount of money that Homer Simpson would spend, $SD(Y)$.

1. Homer Simpson is going to Moe's Tavern for some *Flaming Moe*'s. Let X denote the number of *Flaming Moe*'s that Homer Simpson will drink. Suppose X has the the following probability distribution:

x	$f(x)$	$x\,f(x)$	$x^2 f(x)$
0	0.1	0.0	0.0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.3	0.9	2.7
4	*0.1*	0.4	1.6
	1.0	2.1	5.7

a) Find the missing probability $f(4) = \mathrm{P}(\mathrm{X} = 4)$.

$f(4) = 1 - [\,0.1 + 0.2 + 0.3 + 0.3\,] = \mathbf{0.10}$.

b) Find the probability $\mathrm{P}(\mathrm{X} \geq 1)$.

$\mathrm{P}(\mathrm{X} \geq 1) = \mathbf{0.90}$.

c) Find the probability $\mathrm{P}(\mathrm{X} \geq 1 \mid \mathrm{X} < 3)$.

$$\mathrm{P}(\mathrm{X} \geq 1 \mid \mathrm{X} < 3) = \frac{\mathrm{P}(\mathrm{X} \geq 1 \cap \mathrm{X} < 3)}{\mathrm{P}(\mathrm{X} < 3)} = \frac{0.5}{0.6} \approx \mathbf{0.8333}.$$

d) Compute the expected value of X, $\mathrm{E}(\mathrm{X})$.

$$\mathrm{E}(\mathrm{X}) = \sum_{\text{all x}} \mathrm{x} \cdot f(\mathrm{x}) = \mathbf{2.1}.$$

e) Compute the standard deviation of X, $\mathrm{SD}(\mathrm{X})$.

$$\mathrm{Var}(\mathrm{X}) = \mathrm{E}(\mathrm{X}^2) - [\,\mathrm{E}(\mathrm{X})\,]^2 = 5.7 - (\,2.1\,)^2 = 1.29.$$

$$\mathrm{SD}(\mathrm{X}) = \sqrt{1.29} \approx \mathbf{1.1358}.$$

1. (continued)

Suppose each *Flaming Moe* costs \$1.50, and there is a cover charge of \$1.00 at the door. Let Y denote the amount of money Homer Simpson spends at the bar. Then $\mathrm{Y} = 1.50 \cdot \mathrm{X} + 1.00$.

f) Find the probability that Homer would spend over \$5.00.

x	y	$f(x) = f(y)$
0	\$1.00	0.10
1	\$2.50	0.20
2	\$4.00	0.30
3	\$5.50	0.30
4	\$7.00	0.10
		1.00

$\mathrm{P}(\mathrm{Y} > \$5.00) = \mathrm{P}(\mathrm{X} \geq 3) = \mathbf{0.40}$.

g) Find the expected amount of money that Homer Simpson would spend, $\mathrm{E}(\mathrm{Y})$.

$\mathrm{E}(\mathrm{Y}) = 1.50 \cdot \mathrm{E}(\mathrm{X}) + 1.00 = \mathbf{\$4.15}$.

(On average, Homer drinks 2.1 *Flaming Moe*'s per visit, his expected payment for the drinks is \$3.15. His expected total payment is \$4.15 since he has to pay \$1.00 for the cover charge.)

OR

x	y	$f(x) = f(y)$	$y \cdot f(y)$
0	\$1.00	0.10	0.10
1	\$2.50	0.20	0.50
2	\$4.00	0.30	1.20
3	\$5.50	0.30	1.65
4	\$7.00	0.10	0.70
		1.00	**4.15**

$$\mathrm{E}(\mathrm{Y}) = \sum_{\text{all } y} y \cdot f(y) = \$\mathbf{4.15}.$$

h) Find the standard deviation for the amount of money that Homer Simpson would spend, $\mathrm{SD}(\mathrm{Y})$.

$\mathrm{SD}(\mathrm{Y}) = |\,1.50\,| \cdot \mathrm{SD}(\mathrm{X}) \approx$ **\$1.7037**.

OR

x	y	$f(x) = f(y)$	$y^2 \cdot f(y)$
0	\$1.00	0.10	0.100
1	\$2.50	0.20	1.250
2	\$4.00	0.30	4.800
3	\$5.50	0.30	9.075
4	\$7.00	0.10	4.900
		1.00	20.125

$$\mathrm{Var}(\mathrm{Y}) = \mathrm{E}(\mathrm{Y}^2) - [\mathrm{E}(\mathrm{Y})]^2 = 20.125 - (4.15)^2$$

$$= 20.125 - 17.2225 = 2.9025.$$

$\mathrm{SD}(\mathrm{Y}) = \sqrt{2.9025} \approx$ **\$1.7037**.