STAT 400
UIUC

# More Answers for 2.2, 2.3

Stepanov
Dalpiaz

**5.** Management of an airline knows that 0.5% of the airline's passengers lose their luggage on domestic flights. Management also know that the average value claimed for a lost piece of luggage on domestic flights is $600. The company is considering increasing fares by an appropriate amount to cover expected compensation to passengers who lose their luggage. By how much should the airline increase fares?

Let X denote the compensation to a passenger due to lost luggage.
Then X has the following probability distribution:

| x | $f(x)$ | $x \cdot f(x)$ |
|---|---|---|
| $ 0 | 0.995 | 0 |
| $ 600 | 0.005 | 3 |
| | 1.00 | $ 3 |

The average compensation to each passenger due to lost luggage is $ 3.
The airline should increase fares by $ **3**.

**5½.** A customer for a home insurance policy owns a $200,000 home. The probability is 0.1% that the home will be totally destroyed by fire, and the probability is 0.5% that the home will suffer a 50% loss due to fire. If we ignore all other partial losses, what premium should the insurance company charge for a policy just to break even?

Let X = the amount of compensation paid to the customer.

Then $P(X = 200{,}000) = 0.001$, $P(X = 100{,}000) = 0.005$,

$P(X = 0) = 1 - [0.001 + 0.005] = 0.994$.

Want $E(X) = \sum x \cdot f(x) = ?$

| $x$ | $f(x)$ | $x \cdot f(x)$ |
|---|---|---|
| 0 | 0.994 | 0 |
| 100,000 | 0.005 | 500 |
| 200,000 | 0.001 | 200 |
| | 1.000 | **700** |

The insurance company should charge **$700** for the policy.

**6.** A grocer has shelf space for two units of a highly perishable item that must be disposed of at the end of the day if it is not sold. Each unit costs $1.70 and sells for $3.50. From past experience, the grocer knows that the demand for this item could be either 0, or 1, or 2 units. The grocer has to decide whether to stock one unit of this item or two units.

a) Construct the grocer's profit (payoff) in each of the following cases:

| | demand = 0 | demand = 1 | demand = 2 |
|---|---|---|---|
| stock 1 unit | – 1.70<br>**– 1.70** | – 1.70 + 3.50<br>**1.80** | – 1.70 + 3.50<br>**1.80** |
| stock 2 units | – 3.40<br>**– 3.40** | – 3.40 + 3.50<br>**0.10** | – 3.40 + 7.00<br>**3.60** |

b) Suppose P(demand = 0) = 0.20, P(demand = 1) = 0.30, and P(demand = 2) = 0.50. Find the expected payoff (profit) of both actions and choose the "optimal" action.

Stock 1 unit:

Expected profit = (– 1.70) (0.20) + (1.80) (0.30) + (1.80) (0.50) = **$1.10**.

Stock 2 units:

Expected profit = (– 3.40) (0.20) + (0.10) (0.30) + (3.60) (0.50) = **$1.15**.

Optimal action – **stock 2 units**.

c) Suppose P(demand = 0) = 0.25, P(demand = 1) = 0.30, and P(demand = 2) = 0.45. Find the expected payoff (profit) of both actions and choose the "optimal" action.

Stock 1 unit:

Expected profit = (– 1.70) (0.25) + (1.80) (0.30) + (1.80) (0.45) = **$0.925**.

Stock 2 units:

Expected profit = (– 3.40) (0.25) + (0.10) (0.30) + (3.60) (0.45) = **$0.80**.

Optimal action – **stock 1 unit**.

**6½.** A baker produces a certain type of cake with cost \$10 and sells it for \$15. The cakes are made during the weekend and should be sold during the next week. If a cake is not sold during the week, it is placed on the rack for the reduced items and then is eventually sold for \$3. The weekly demand follows the distribution:

| d | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| P(d) | 0.15 | 0.30 | 0.35 | 0.20 |

How many cakes should the baker make?

Payoff (profit) in dollars:

| | d = Demand | 7 cakes | 8 cakes | 9 cakes | 10 cakes |
|---|---|---|---|---|---|
| Action | Number of cakes made | | | | |
| $a_1$ | 7 | **35** | **35** | **35** | **35** |
| $a_2$ | 8 | **28** | **40** | **40** | **40** |
| $a_3$ | 9 | **21** | **33** | **45** | **45** |
| $a_4$ | 10 | **14** | **26** | **38** | **50** |

| Action | Expected Payoff |
|---|---|
| $a_1$ | **35**. |
| $a_2$ | $28 \times 0.15 + 40 \times 0.85 =$ **38.2**. ← largest |
| $a_3$ | $21 \times 0.15 + 33 \times 0.30 + 45 \times 0.55 =$ **37.8**. |
| $a_4$ | $14 \times 0.15 + 26 \times 0.30 + 38 \times 0.35 + 50 \times 0.20 =$ **33.2**. |

The best decision based on expected payoff is $a_2$ = make **8** cakes.

**7.** To save time and money when testing blood samples for the presence of a disease, the following procedure is sometimes used: Blood samples from a number of people are pooled together and the pooled sample is tested. If the pooled sample is negative, then no further testing needs to be done. If the pooled sample is positive, however, then each one of the separate samples must be tested individually. To be specific, suppose that the blood samples of 10 people are pooled together in this procedure, and suppose that the proportion of those who have the disease in the population from which the samples are drawn is 0.02. (Also assume that the presence of the disease in one sample is independent of the presence of the disease in the other samples.)

a) Find the probability that the pooled sample is negative.

Blood samples of 10 people are pooled together.
Proportion of those who have the disease in the population is 0.02.
Samples are independent.

P( the pooled sample is negative ) = P( all 10 individual samples are negative )

$$= P(\text{1st} - \cap \text{2nd} - \cap \ldots \cap \text{10th} -)$$

by independence

$$= P(\text{1st} -) \cdot P(\text{2nd} -) \cdot \ldots \cdot P(\text{10th} -) = (0.98)^{10} = \mathbf{0.817}.$$

b) Let $X$ be the number of tests that must be run. Construct the probability distribution of $X$.

$X$ = the number of tests that must be run.

If the pooled sample is negative, then only one test is needed.

If the pooled sample is positive, then 11 tests are needed (1 for the pooled sample and 10 for each individual sample)

| x | $f(x)$ |
|---|---|
| 1 | 0.817 |
| 11 | 0.183 |
| | 1.000 |

c) Compute the mean of $X$. Is this larger or smaller than 10?

| x | $f(x)$ | $x \cdot f(x)$ |
|---|---|---|
| 1 | 0.817 | 0.817 |
| 11 | 0.183 | 2.013 |
| | 1.000 | **2.830** |

"On average," **2.830** tests would be needed (per 10 individuals).